CHAPTER 2

GEOMETRIC CONSTRUCTION

Overview

Introduction

Familiarity with the step-by-step methods used for constructing geometric figures and knowing related definition of terms help you understand the practical applications of geometric construction to problem solving. Simplified- or preferred methods of geometric construction, as well as alternate methods, are valuable knowledge factors when used with drafting instruments to create accurate drawings. Geometric construction applies equally to computer-generated drawings as it does to more traditional instrumental drawings using triangles, compasses, protractors, and straightedges.

Objectives

The material in this chapter enables you to do the following:

- Identify angles as acute, obtuse, complimentary, or supplemental.
- Inscribe and circumscribe geometric figures.
- Calculate the degrees in an angle of a regular polygon.
- Identify quadrilaterals.
- Bisect lines, angles, and circles.
- Create an ellipse using the trammel or foci method.

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Acronyms

The following table contains a list of acronyms you must know to understand the material in this chapter:

Acronym	Meaning
DIA	Diameter
PI or π	3.1416
RAD or R	Radius

In this chapter

This chapter covers the following topics:

Definition of Terms

Introduction

Knowing the professional terminology used in a selected field is imperative for any degree of success. Illustrator Draftsman is no different. Without a solid knowledge of the definition of terms used in this field, you will be lost. Make sure you understand this chapter before you skip ahead to other chapters in this training manual.

Points and lines

This basic discussion on points and lines is to make sure you understand their definition and representation. If you do not understand how to succinctly draw a point or line, a viewer will not know how to interpret them.

POINT: A point is a location in space. It has no height, width, or depth. Represent a point of intersection on lines with short crossbars or between two lines with short cross hairs. Do not represent a point with a mere dot on paper.

Figure 2-1 shows how to represent points on paper.

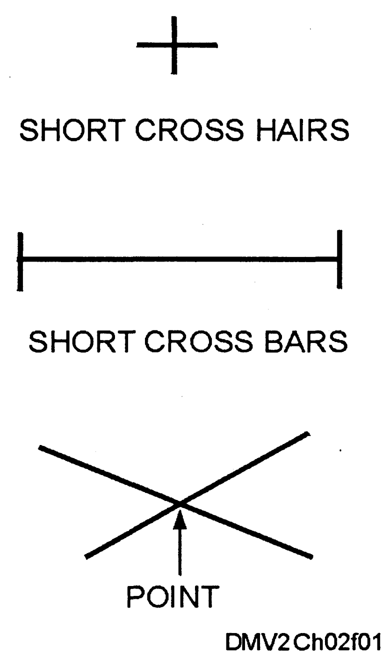

Figure 2-1.—Points.

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Definition of Terms, Continued

Points and lines (Continued)

LINE: A *straight line* is the shortest distance between two points and is often referred to simply as a line. If the length of a line is indefinite or without fixed endpoints, its length is any distance you select. If the line has fixed endpoints, mark them with small mechanically drawn cross hairs. Straight or curved lines are *parallel* if the shortest distance between them remains constant throughout their length. If a line is parallel to another line, use the common symbol for parallelism (||). Lines that intersect at right angles (90°) to each other are referred to as *perpendicular*. Indicate perpendicularity with short lines intersecting at right angles (⊥ (singular) or ⊥s (plural)) or a small square box at the apex (∟) of the intersection.

Figure 2-2 shows common line terminology.

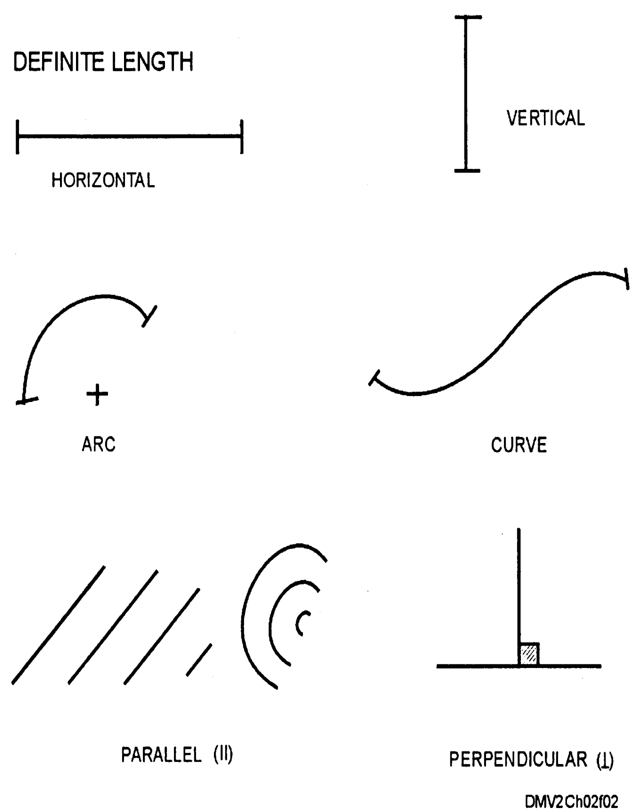

Figure 2-2. —Lines.

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Angles

Angles form when two lines intersect. The symbols for angularity are ∠ (singular) or ∠s (plural). There are a maximum of 180 possible degrees to an angle. A *straight angle* is an angle of 180° and appears as a straight line. *Obtuse angles* are angles less than 180° but more than 90°. An angle of 90° is referred to as a *right angle* because of the relationship between the two intersecting lines. *Acute angles* are angles less than 90°. When two angles are combined to total 90°, they are referred to as *complimentary angles. Supplementary angles* form when two angles combine to total 180°. You may draw angles at any degree of angularity using triangles or a protractor. To increase accuracy, use a vernier protractor or construct angles using the tangent, sine, or chord methods.

Figure 2-3 illustrates the different degrees of standard angularity.

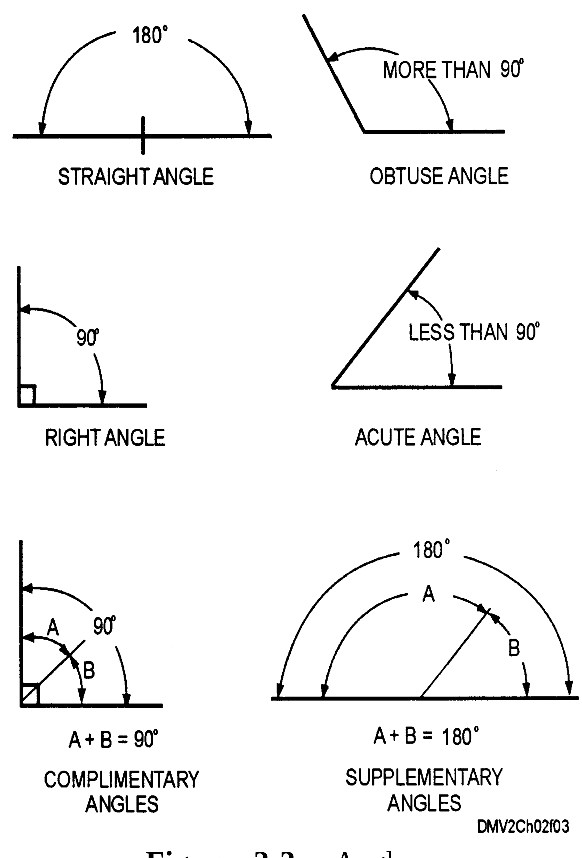

Figure 2-3.—Angles.

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Triangles

A *triangle* is a plane figure bound by three straight sides, which form three interior angles. The top of the triangle is the *vertex.* The height of a triangle is referred to as the *altitude.* The bottom of a triangle is its *base.* The sum of the three interior angles is always 180°. When all sides and all interior angles (60°) are equal, the triangle is referred to as an *equilateral triangle.* When two sides and two angles are equal, the triangle is an *isosceles triangle.* A *scalene triangle* does not have any equal sides or angles. A *right triangle* has one angle equal to 90° and the long side opposing that angle is called the *hypotenuse.*

Figure 2-4 shows triangles.

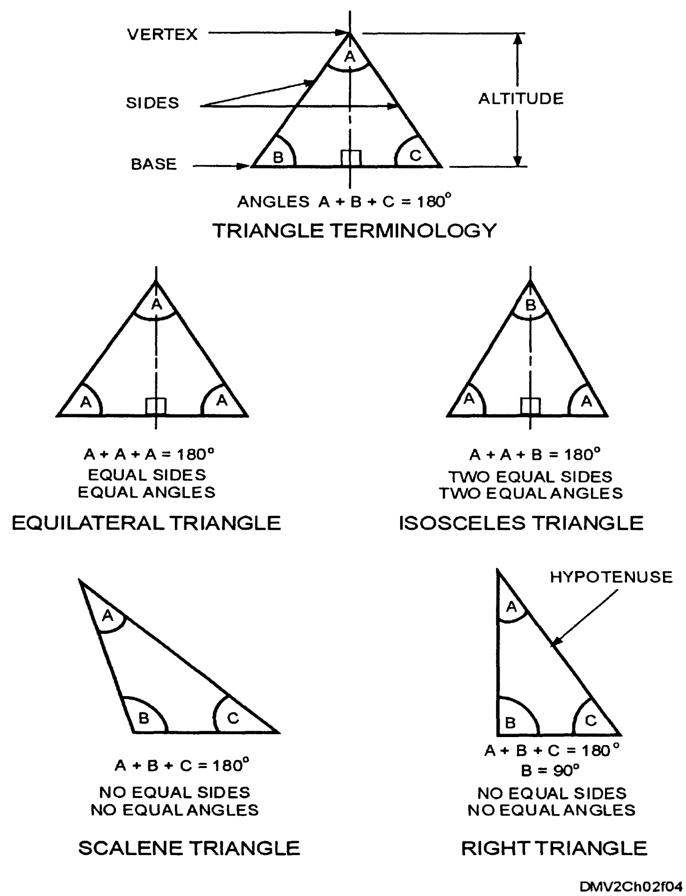

Figure 2-4.—Triangles.

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Quadrilaterals

Quadrilaterals are plane figures bound by four straight sides. Four-sided figures with parallel opposing sides are further classified as *parallelograms.* A parallelogram having four equal sides and equal angles is called a *square.* A *rhombus* has four equal sides and equal opposing angles. A figure with equal opposing sides and equal angles is a *rectangle.* A figure with equal opposing sides and equal opposing angles is a *rhomboid.* Quadrilaterals with only two parallel sides and no angles equal are *trapezoids.* If no sides and no angles are equal or parallel, the figure is called a *trapezium.* Trapezoids and trapeziums are quadrilaterals but are not parallelograms.

Figure 2-5 shows the six different types of quadrilaterals.

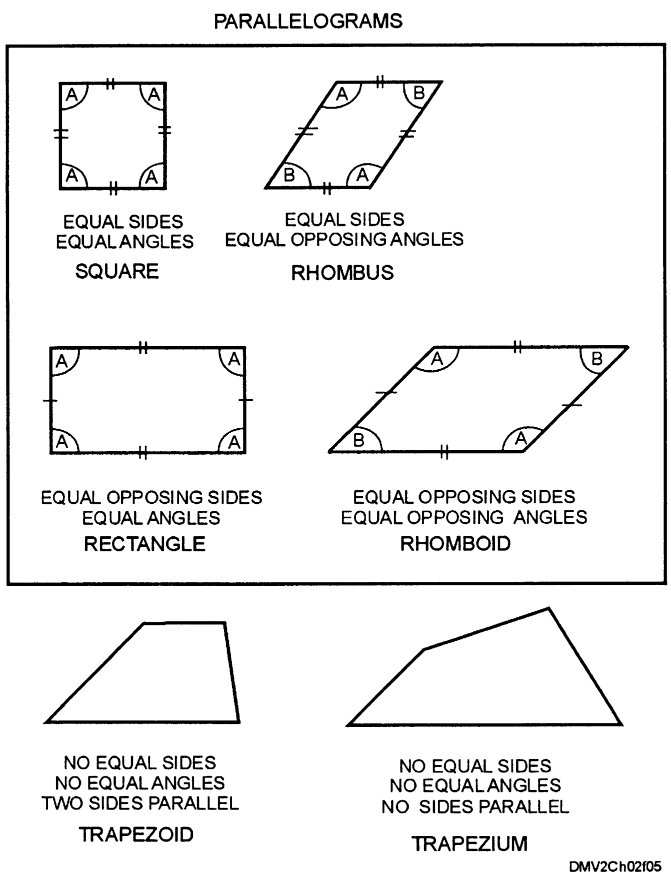

Figure 2-5.—Quadrilaterals.

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Polygons

Any plane figure bound by straight sides is a *polygon.* This definition includes triangles and quadrilaterals. Polygons having equal sides and equal angles are called *regular polygons* (including equilateral triangles and squares) and can be constructed by inscribing in or circumscribing around a circle or square, a technique covered later in this chapter. The following list shows how the names of the regular polygons change with the number of sides:

Sides	Name
3	Triangle
4	Square
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon

Figure 2-6 shows regular polygons.

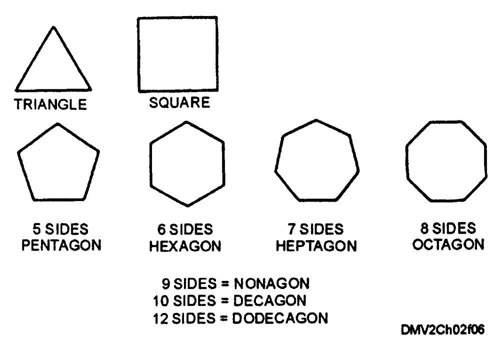

Figure 2-6.—Regular polygons.

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Circles

A *circle* is a closed curve in which all points along the curve are equidistant from the center. The distance from the center point to any point along the circle edge is called a *radius* (RAD or R). The distance from one side of the circle through the center point to the opposing side of the circle is the circle *diameter* (DIA). Half of the distance around a circle is called a *semicircle. Circumference* refers to the total distance around the circle. Calculate the circumference of a circle by multiplying the diameter of the circle by 3.1416 or π (pronounced pi). A *chord* is a straight line joining two points on a curve. A *segment* is the section of the curve cut off by the line or chord. *Quadrants* result from the intersection of two radii at 90° including the portion of the circle between the radii. *Sectors* are the part of the circle bound by two radii at other than right angles including the bound portion of the circle. *Angles* are formed by the intersection of radii but do not include the bound portion of the circle. An *arc* is a segment of the curved portion of the circle bound by the intersection of two radii but does not include the radii. A straight line that intersects and passes through two points on the circle is called a *secant.* Straight lines that touch but do not intersect at one point on a circle are said to be *tangent.* Multiple circles sharing a common center point are called *concentric circles.* Multiple circles that do not share a common center point are referred to as *eccentric circles.* Eccentric circles are most common in depicting reciprocal relationships such as in the camshaft of an engine.

Figure 2-7 illustrates circle terminology.

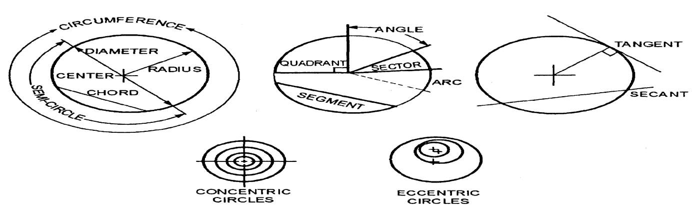

Figure 2-7.—Circle terminology.

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Circles (Continued)

Circles are also used to construct curves. The *Spiral of Archimedes* is a curve that forms at a fixed point in the center of the circle and rotates through geometrically determined points or locus. As the locus uniformly increase or decrease their distance from the center, the spiral emerges.

Figure 2-8 shows a Spiral of Archimedes.

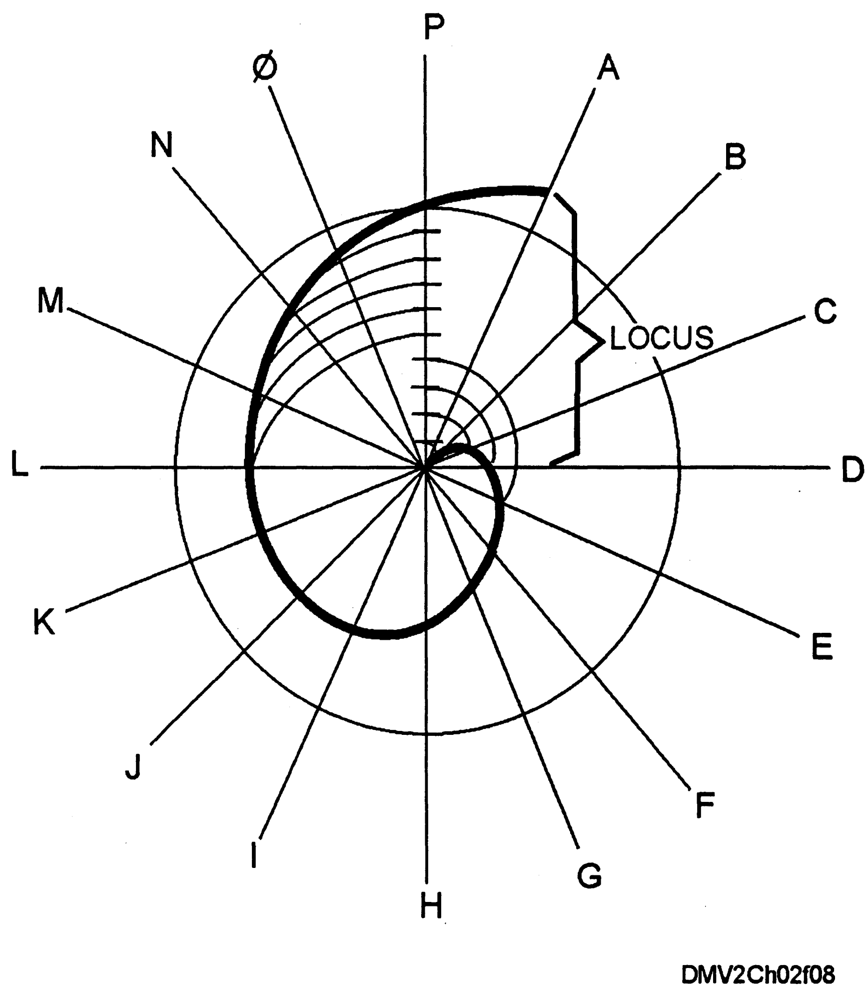

Figure 2-8.—The Spiral of Archimedes.

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Circles (Continued)

Points along the circumference of a circle rolling on a straight line are known as cycloidal or *cycloids.* Points along the circumference of a circle rolling on the convex side or outside edge of an equal or larger circle are called *epicycloid.* If the circumference of a circle rolls along the concave side or inside edge of a larger circle, the resulting curve is a *hypocycloid.*

Figure 2-9 illustrates the formation of a cycloid, epicycloid, and hypocycloid.

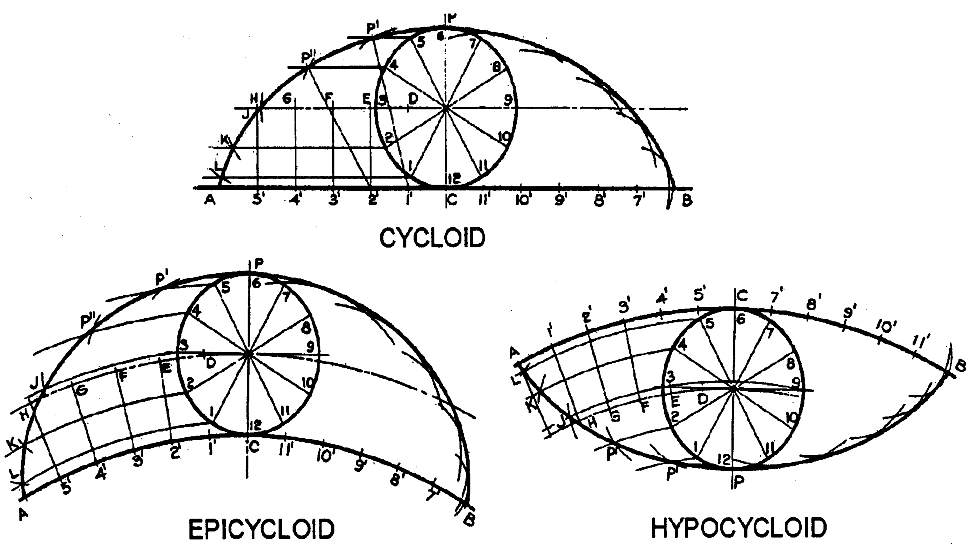

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Figure 2-9.—Cycloids, epicycloids, and hypocycloids.

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Solids

Solids are figures having the three dimensions of length, width, and depth bounded by plane surfaces. Solids may also be known as *polyhedra*. The plane surfaces of polyhedra are called *faces* and if the faces are regular polygons, the solids are *regular polyhedra*.

Figure 2-10 shows regular polyhedra or solids.

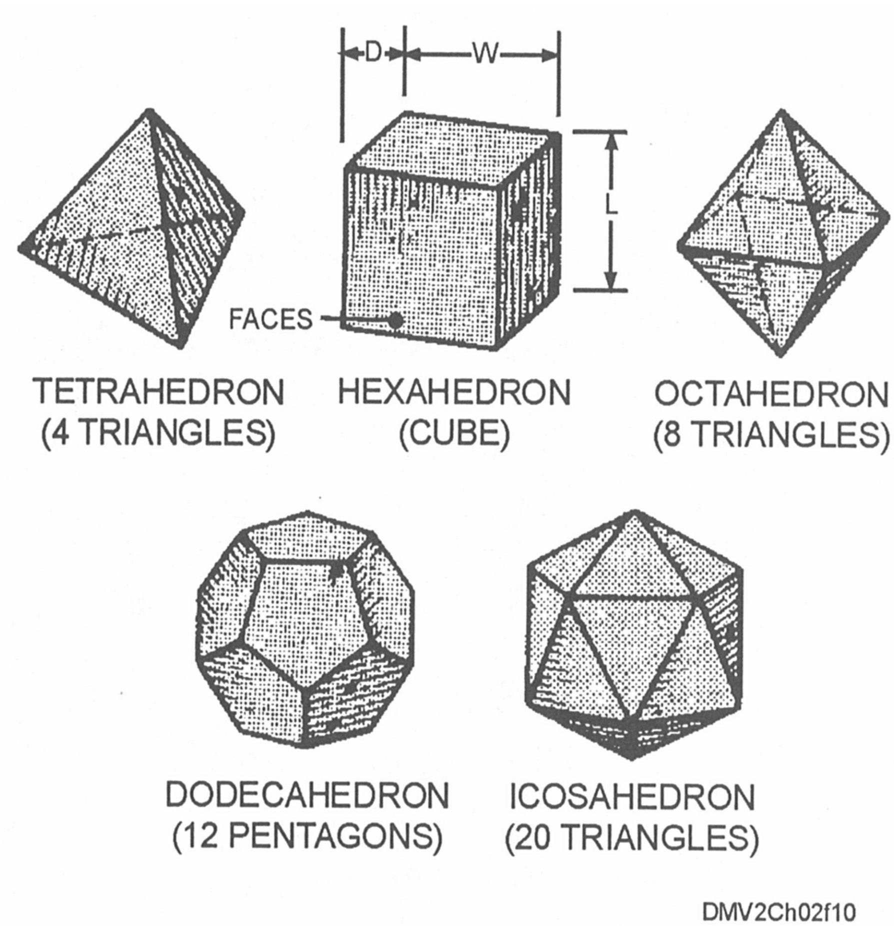

Figure 2-10.—Regular polyhedra.

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Solids (Continued)

A *prism* is a solid with two bases (top and bottom) that are equal regular polygons and three or more lateral faces that are parallelograms. If the bases are also parallelograms, the prism is a *parallelepiped.* A *right prism* has faces and lateral edges that are perpendicular to the bases. *Oblique prisms* have faces and lateral edges oblique to the bases.

Figure 2-11 illustrates prism configurations.

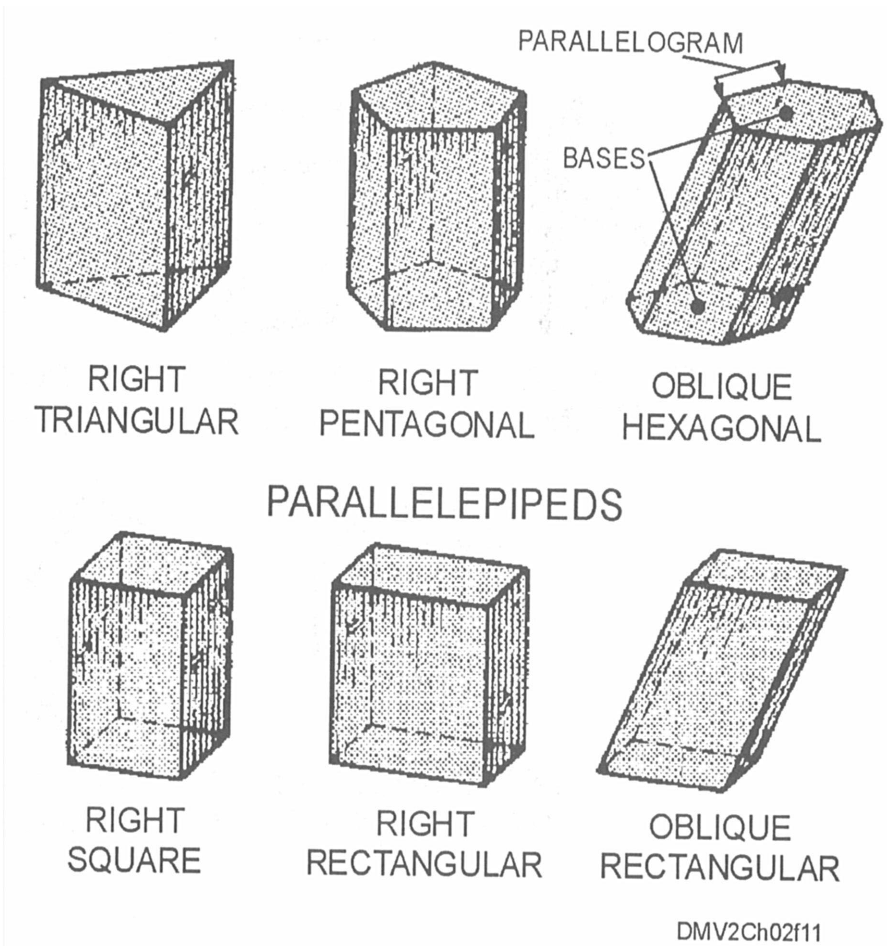

Figure 2-11.—Prisms.

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Solids (Continued)

Cylinders are two parallel bases formed by a fixed curve or *directrix* revolving around a straight line or *generatrix* at the center. The generatrix at the center of the cylinder is also called an *axis.* The height of the cylinder is called the altitude. Any point along the edges of the cylinder is referred to as an *element. Right circular* cylinders have lateral edges perpendicular to the bases and *oblique circular* cylinders have lateral edges oblique to the bases. Moving a point around and along the surface of a cylinder with uniform angular velocity to the axis and with a uniform linear velocity in the direction of the axis produces a *helix.* You may construct a helix using a cylinder or cone.

Figure 2-12 show examples of cylinders and a helix.

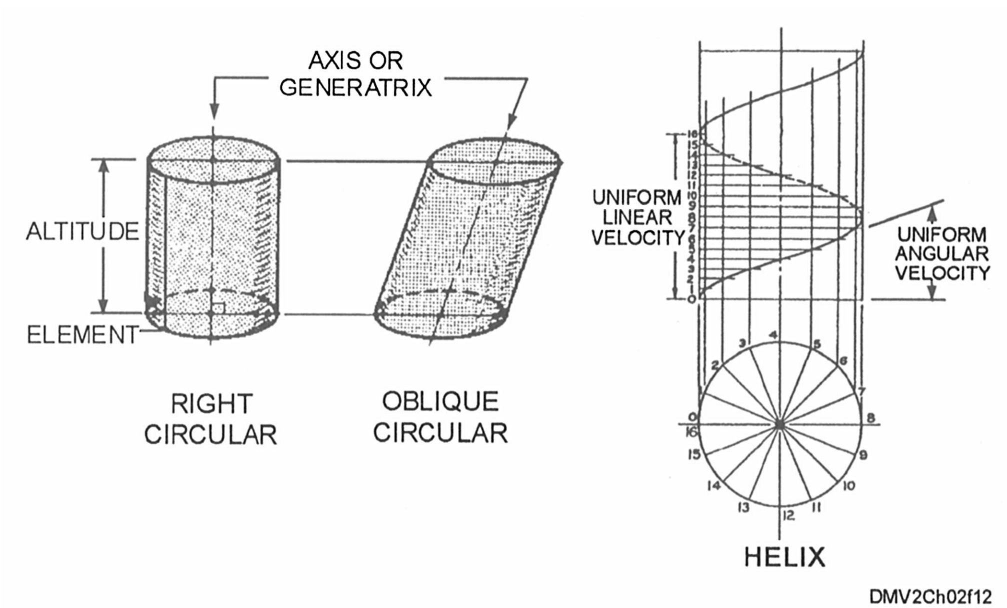

Figure 2-12.—Cylinders and a helix.

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Solids (Continued)

Pyramids have polygons for a base and triangular lateral faces that intersect at the vertex or top of the pyramid. A centerline from the vertex to the center of the base is known as the axis and its height is called the altitude. If the axis is perpendicular to the base, the pyramid is a *right pyramid.* All other pyramids are *oblique pyramids.* A pyramid that has been cut off near the vertex oblique to the base is said to be *truncated;* if the pyramid is cut off parallel to the base, the cut plane is known as a *frustum.*

Figure 2-13 illustrates pyramid terminology.

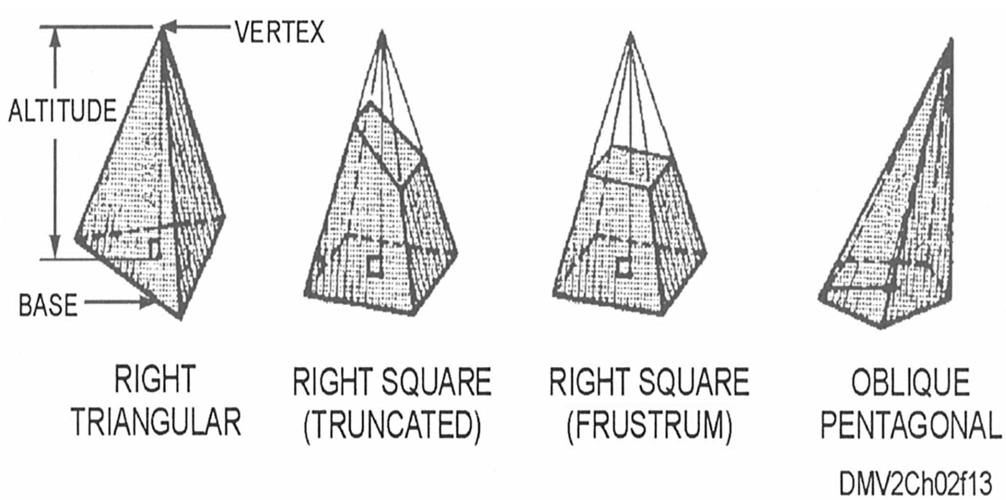

Figure 2-13.—Pyramids.

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Solids (Continued)

Cones have a generatrix that terminates in a fixed point at a vertex around which revolves a directrix or closed curve base. The generatrix is also known as the axis whose height is referred to as altitude. Any point around the cone from the base to the vertex is called an element. A cone whose axis is perpendicular to its base is a *right cone.* Planes intersecting a cone will make the cone appear truncated or frustum. Planes intersecting a right cone produce *conic sections.* Conic sections appear as curves. A conic section perpendicular to the axis appears as a circle at the plane of intersection. A conic section with a cutting plane oblique to the axis but making a greater angle with the axis than the elements appears as an ellipse. When the plane of intersection is oblique to the axis and at the same angle to the axis as the elements, the curves is referred to as a *parabola.* An oblique plane of intersection that makes a smaller angle to the axis than the elements is known as a *hyperbola.* Cones may also be used to construct helixes.

Figure 2-14 shows cones.

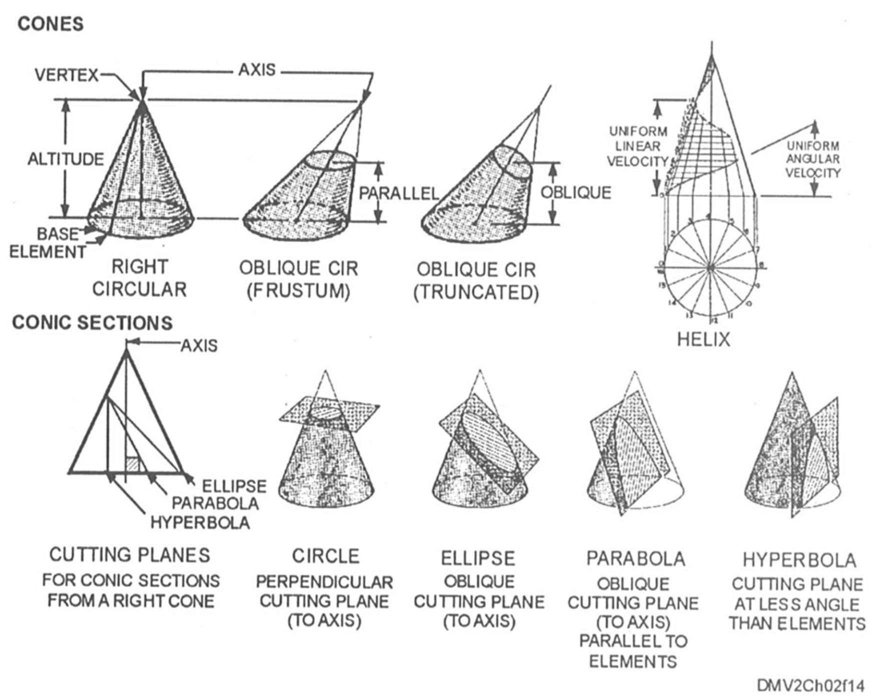

Figure 2-14.—Cone terminology.

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Solids (Continued)

Spheres are formed by a circle revolving around its diameter. The diameter of the circle then becomes the axis and the ends of the axis are known as *poles*.

Figure 2-15 shows a sphere, its axis, and its poles.

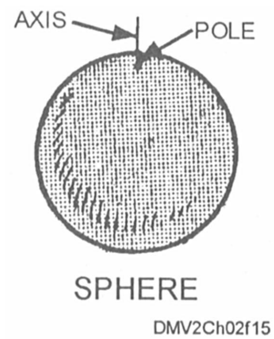

Figure 2-15.—A sphere.

A *torus* or *toroid* is formed by a circle or curve revolving around but not intersecting or containing an axis in its own plane. The axis of a torus is eccentric to the diameter of the circle or curve.

Figure 2-16 is an example of a torus.

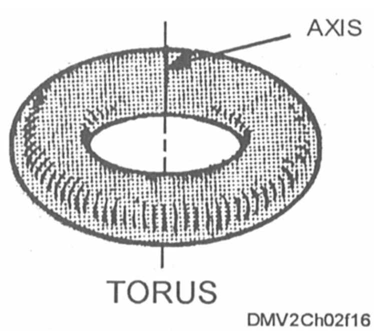

Figure 2-16.—A torus.

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Definition of Terms, Continued

Solids (Continued)

Ellipsoids are geometric surfaces whose plane sections are all ellipses or circles. *Oblate ellipsoids* have flattened surfaces at the poles. Ellipsoids flattened so that the altitude of the polar axis exceeds the equatorial diameter are called *prolate ellipsoids.*

Figure 2-17 shows the polar distortions of ellipsoids.

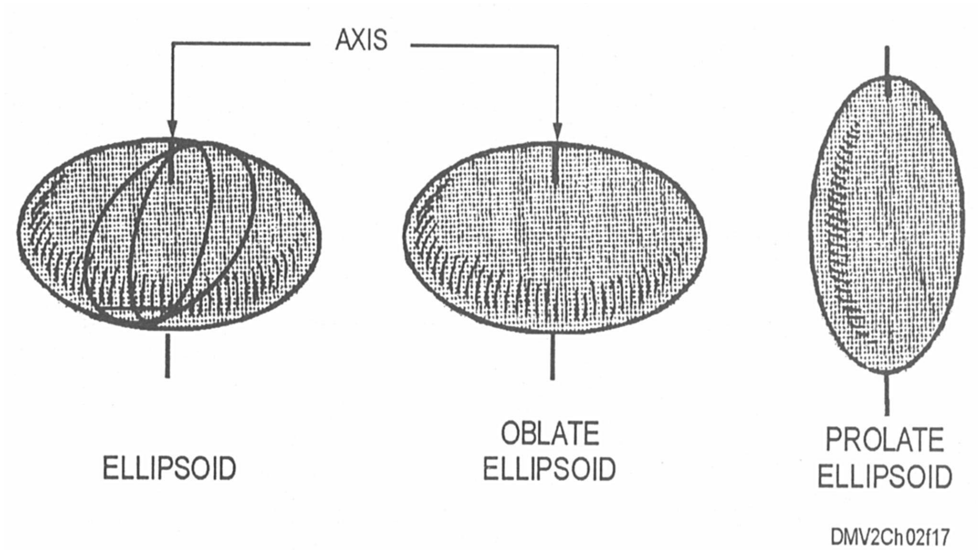

Figure 2-17.—Ellipsoids.

Bisection

Introduction

Problem solving with geometric constructions often involves dividing a given entity. Dividing geometric figures into two equal parts is called *bisecting.* The line that bisects the figure is known as a *bisector. You* should know how to bisect different geometric figures to accurately solve drafting problems.

Bisecting lines or circular arcs

To bisect a line or arc, use this table:

Step	Action
1	With a given arc or line (AB), use a compass set at a distance greater than half the distance of the given line and draw equal arcs above and below the given line.
2	Use a straightedge to join the intersections of the arcs. This straight line locates the center of the given line or arc.

Figure 2-18 shows how to bisect a line or arc.

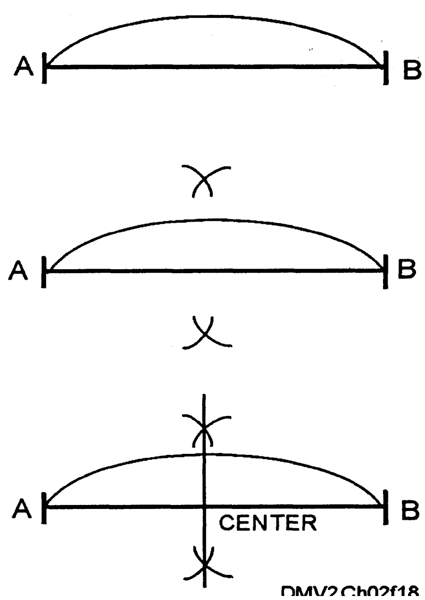

Figure 2-18.—Bisecting a line or arc.

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Bisecting angles To bisect an angle, use this table:

Step	Action
1	Given angle ABC, use a compass to draw an arc at any convenient radius from the apex.
2	Using the compass set at slightly more than half the distance from A to C and with the compass points at the intersections of the arc and angle legs (E and F), draw two short arcs that intersect at D.
3	Draw a straight line from A to D. This bisector divides the original given angle into two equal angles.

Figure 2-19 shows how to bisect an angle.

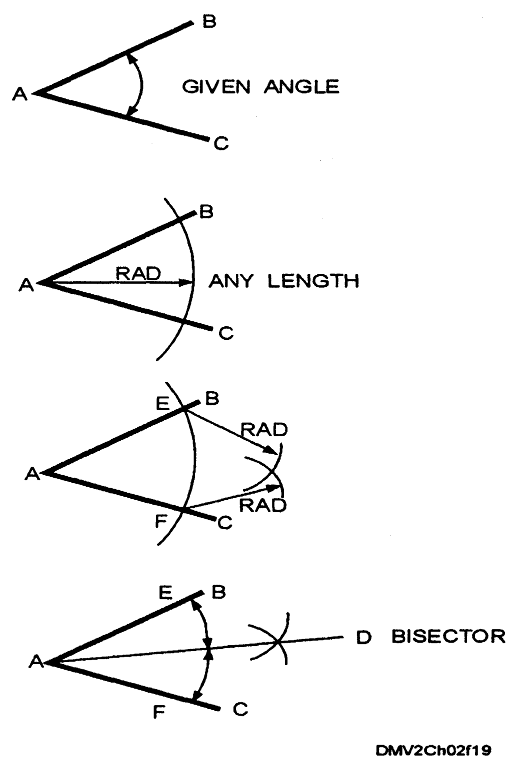

Figure 2-19.—Bisecting angles.

Division

Introduction

When you want two unequal portions or more than two equal portions of a line, you are dividing not bisecting. One of the most frequently executed geometric constructions is dividing a line into multiple equal or proportional parts.

Preferred method for dividing lines into equal parts

To divide a line into equal parts, use this table:

Step	Action
1	Given line AB, draw a perpendicular line (BC).
2	Place a scale or ruler with the first increment (0) at A.
3	Place the desired increment on the scale at the termination point of the perpendicular line (BC).
4	Make tiny pencil marks indicating the desired measurements along the scale.
5	Draw vertical construction lines perpendicular to the given line and parallel to each other. This divides the line into multiple equal increments.

Figure 2-20 shows the steps for the preferred method of dividing a line into multiple equal increments.

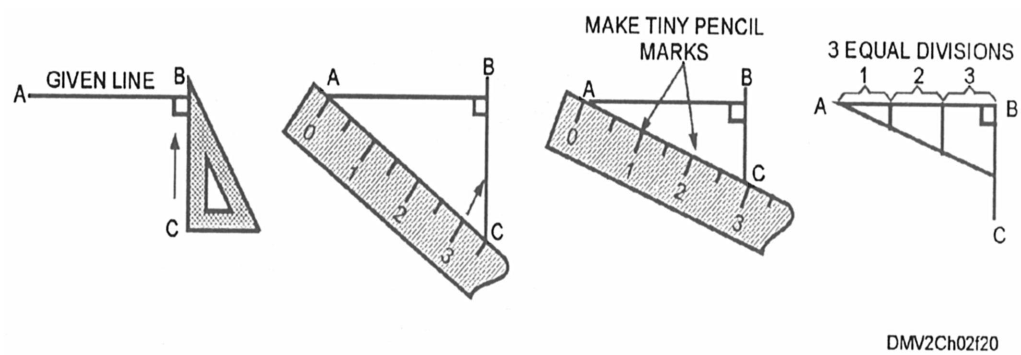

Figure 2-20.—Dividing a line into equal parts.

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Alternate method for dividing lines into equal parts

For an alternate method of dividing a line into equal parts, use this table:

Step	Action
1	Given line AB, draw a light line at any convenient acute angle from A.
2	Set off as many equal divisions as you require along the angled line with a scale or with a pair of dividers.
3	Connect the last increment with the end of the given line (B) with a triangle.
4	Using the same triangle, draw lines from the incremental points to the given line keeping all lines parallel to each other.

Figure 2-21 shows an alternate method for dividing lines into equal segments.

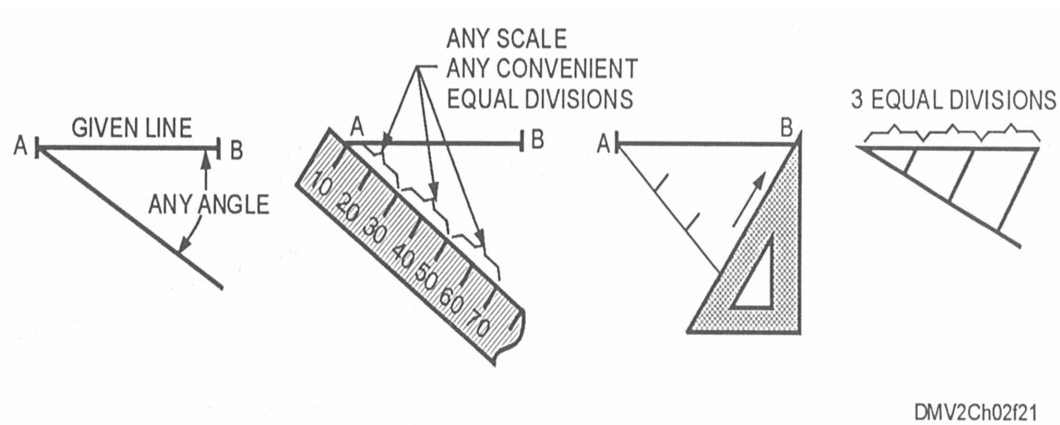

Figure 2-21.—An alternate method of dividing lines.

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Preferred method for dividing lines into proportional parts

To divide a line into proportional parts, use this table:

Step	Action
1	Given line AB, draw a perpendicular line at B.
2	Select any scale and set the first increment (0) at A.
3	Position the scale so that the total number of increments aligns with the perpendicular line at B.
4	Set off the desired increments with tiny pencil marks. In this case 2, 3, and 4 units are marked.
5	Draw vertical lines through these points to the given line.

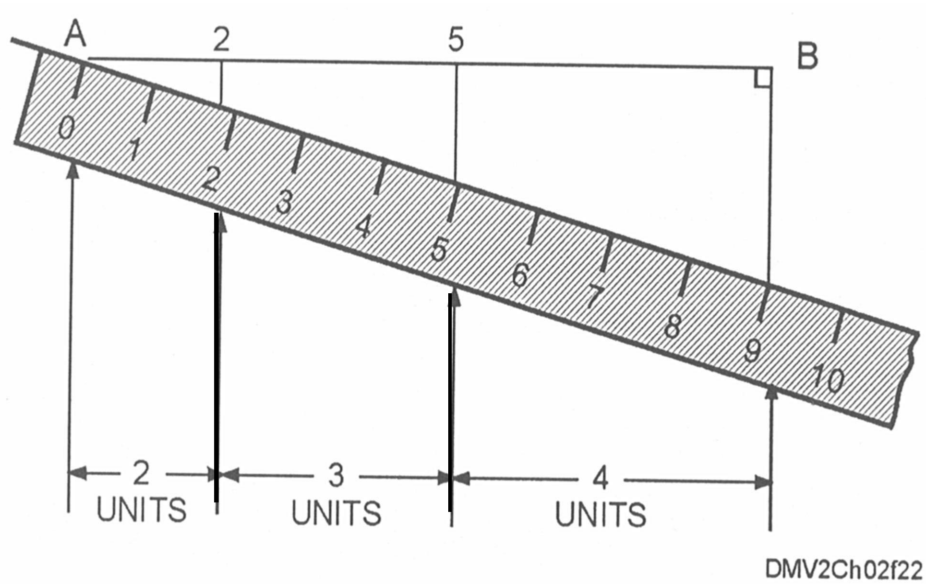

Figure 2-22.—Dividing lines proportionately.

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An alternate method of proportionally dividing lines into parts

For an alternate method of proportionally dividing lines, use this table:

Step	Action
1	Given line AB, draw another line (CD) at any distance below and parallel to the given line.
2	Set your scale along CD with 0 at endpoint C and set off the desired number of increments. Here the increments are 0, 2, 5, and 9.
3	With a straightedge, draw lines through endpoints A and C (0) and B and D(9) to intersect (0) any distance above AB.
4	Using your straightedge, connect lines 2 and 5 to 0.

Figure 2-23 is an alternate method for proportionally dividing lines.

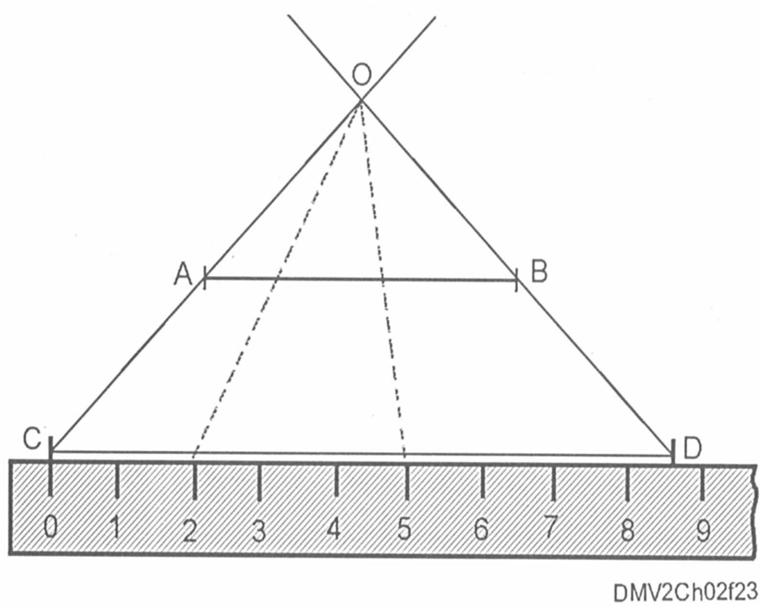

Figure 2-23.—An alternate method of dividing a line proportionately.

Transference

Introduction

At times you may need to move a geometric figure from one location to another on the same or different drawing paper. Transfer polygons, and irregular figures by first drawing a triangle, square, rectangle, or circle around the figure. You should know how to accurately transfer figures.

Transferring angles

To transfer an angle, use this table:

Step	Action
1	Given angle ABC, lay off a line equal to line AB at the new location on the same or different drawing paper (A’, B’).
2	Use any convenient radius (R, R’) and draw arcs using A and A’ as centers.
3	Where the arcs intersect lines AB and A’B’, draw two arcs (r, r’) equal to the distance between A and C.
4	Draw a straight line from A and A’ to the intersection of the two arcs R’ and r’.

Figure 2-24 illustrates how to transfer angles.

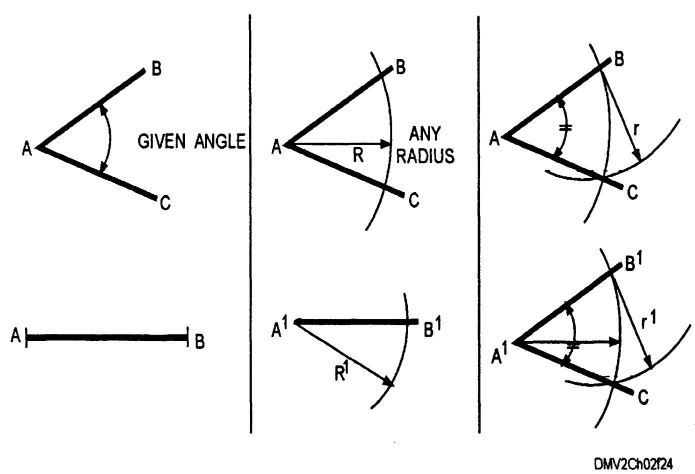

Figure 2-24.—Transferring angles.

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Transferring triangles

To transfer triangles to another location or drawing surface, use this table:

Step	Action
1	Given triangle ABC, set off any side (A’B’) in the new location.
2	Set the compass to the distance of line AC. Place your compass point at A’ and strike an arc.
3	Set the compass for the distance between AB. Place your compass point at B and strike an arc to intersect with the arc drawn from C’.
4	Draw straight lines from A’ to C’ and B’ to C’.

Figure 2-25 shows how to transfer a triangle.

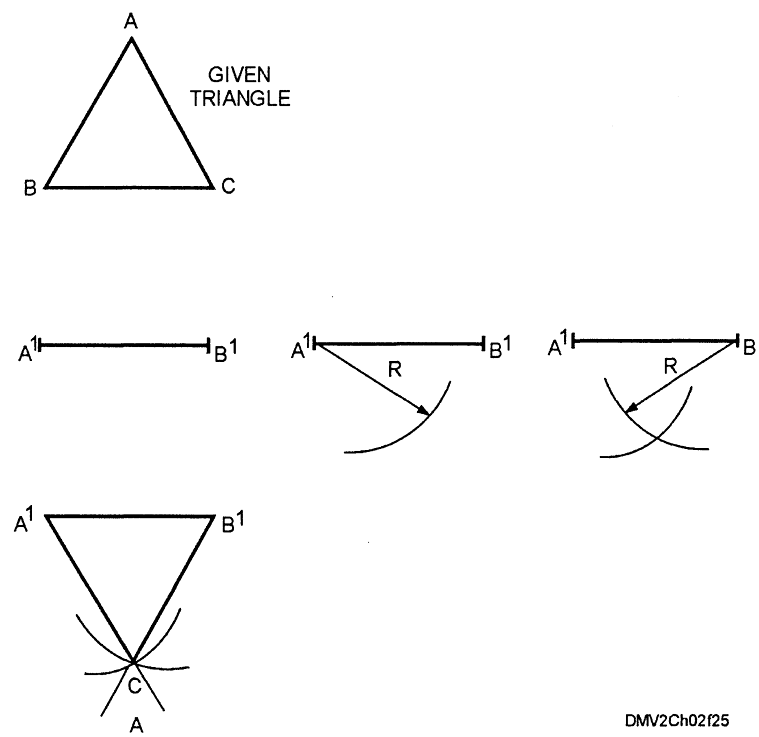

Figure 2-25.—Transferring triangles.

Tangency

Introduction

Tangent lines, arcs, circles, or surfaces are lines, arcs, circles, or surfaces that touch but do not intersect. When drawing irregular or noncircular curves with french curves, you plot a series of tangent arcs. When you indicate round corners on an otherwise straight plane, you are working with an arc that is tangent to two lines at right angles. Make sure all points of tangency are clearly defined before you begin inking.

To draw a circle tangent to a line

To draw a circle tangent to a line at a given point, follow this table:

Step	Action
1	Given line AB with point P representing the point of tangency, erect a perpendicular.
2	Set off the length of the radius of the required circle as a point on the perpendicular and mark it C.
3	Draw the circle using C as the center point and CP as the radius.

Figure 2-26 shows how to draw a circle tangent to a line.

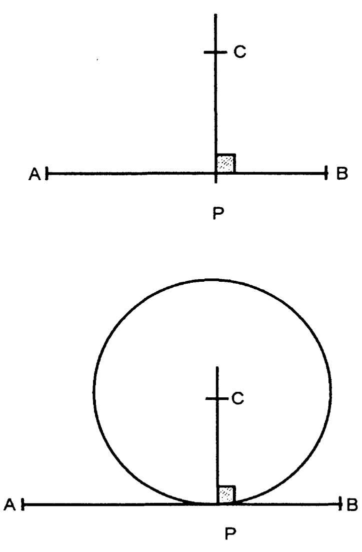

Figure 2-26.—A circle tangent to a line.

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To draw a line tangent to a circle through a point

To draw a tangent to a circle through a point. follow this table:

Step	Action
1	Given a circle with the intended point of tangency at P, move a triangle to a position where one side passes through the center of the circle and point P.
2	Slide the triangle across the straightedge until the opposite side of the triangle touches the circumference at P. If P is outside of the circle, place the triangle with one straight side passing from P to the circumference of the circle. Move the triangle over so that the opposing straight side passes through the center of the circle and intersects the circumference of the circle. Mark this T for point of tangency. Return the triangle to the first position.
2	Draw the required tangent.

Figure 2-27 shows the process for drawing a line tangent to a circle through a given point.

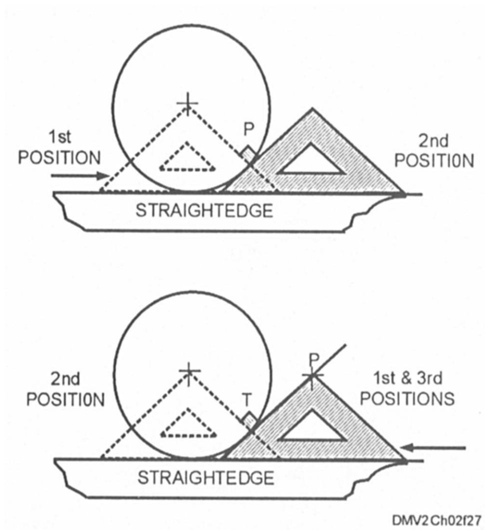

Figure 2-27.—A line tangent to a circle.

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To draw tangents to two circles

To draw tangents to two circles, follow this table:

Step	Action
1	Given two circles with centers marked C1 and C2, move a triangle or straightedge to connect the centerlines.
2	Place a triangle at the upper arcs of the circles until one side of the triangle touches both circles C1 and C2. Draw a line tangent to the circles.
3	Draw a perpendicular line from the points of tangency to the centerlines of the circles.
4	Repeat this procedure for the bottom arcs of the two circles.

Figure 2-28 shows how to draw tangents to two circles.

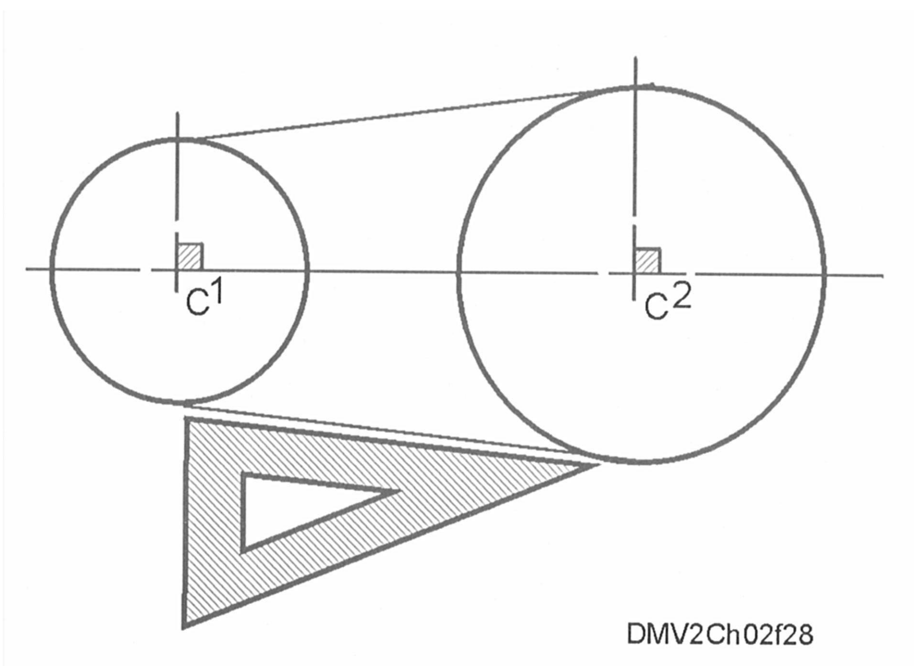

Figure 2-28.—Drawing tangents to two circles.

Continued on next page

To draw an arc tangent to two lines at right angles

It is impractical to draw small radii arcs by tangency construction. For small radii or radii up to 5/8ths inch, draw a 45° bisector of the angle and locate the arc by trial and error. You may also use a circle template so long as the diameter of the circle precisely equals twice the required radius.

To draw an arc tangent to two lines at right angles. follow this table:

Step	Action
1	Given two lines at right angles to each other, strike an arc at a selected radius intersecting the lines at tangent points T.
2	With the same selected radius and using points T as centers, strike another arc to intersect at a point C.
3	With C as a center, use the selected radius to draw the required tangent arc.

Figure 2-29 shows the process for drawing an arc tangent to two lines at right angles.

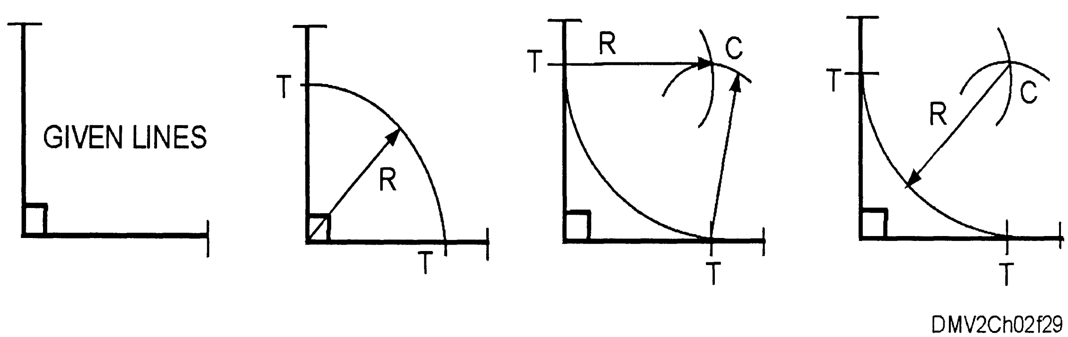

Figure 2-29.—Drawing an arc tangent to a right angle.

Continued on next page

To draw an arc tangent to two lines at acute or obtuse angles

To draw an arc tangent to two lines at acute or obtuse angles, follow this table:

Step	Action
1	Given two lines not at right angles (either greater than (obtuse) or less than (acute) 90°), draw lines parallel to the given lines at a distance that equals the desired radius of the required arc.
2	At the intersection of the parallel lines (C), draw perpendicular lines to locate tangent points T.
3	With C as the center and R as the radius of the required arc, draw the required arc between the points of tangency.

Figure 2-30 shows a tangent arc between acute and obtuse angles.

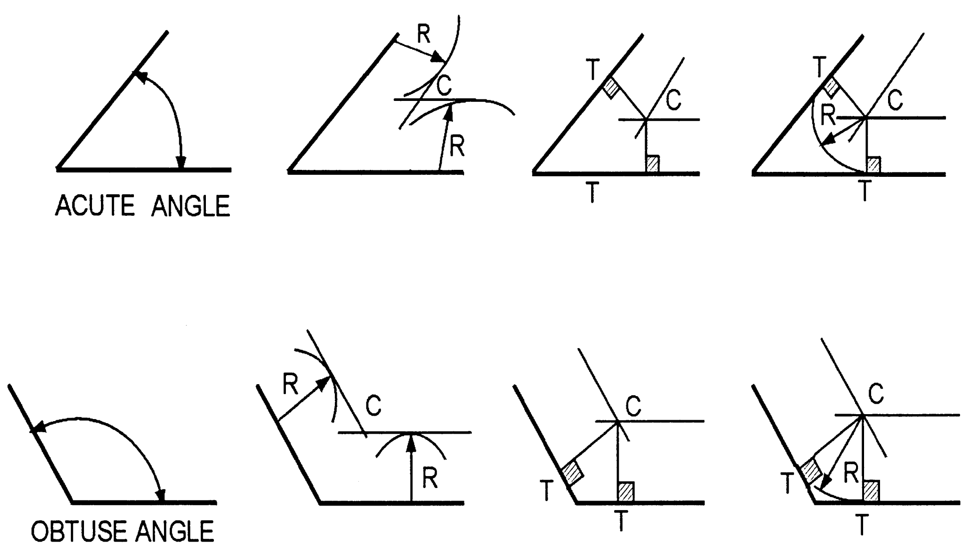

Figure 2-30.—Tangent arcs for acute and obtuse angles.

Continued on next page

To draw an arc tangent to two arcs

To draw an arc tangent to two arcs, follow this table:

Step	Action
1	Given arcs with centers A and B, and a required radius R, draw arcs parallel to the given arcs and at a distance that equals R.
2	Label the intersection of these arcs C since this is the center of the required tangent arc.
3	Draw lines from the centers A and B to C to locate points of tangency T.
4	Draw the required arc with a radius of R to the points of tangency.

Figure 2-31 shows the process of drawing a tangent arc to two arcs.

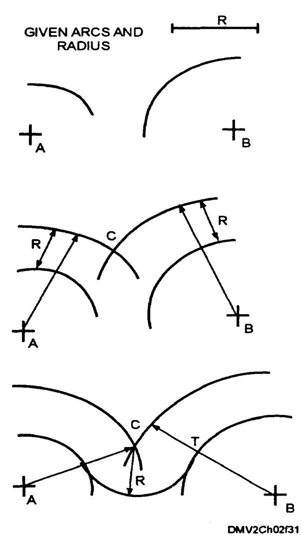

Figure 2-31. —Drawing an arc tangent to two arcs.

Clearly this section does not cover all situations of tangency but shows that the problems have mathematical solutions. Refer to texts in basic drafting for more complete coverage of tangential problem solving.

Polygonal Construction

Introduction

Polygons are the most common geometric constructions. A thorough understanding of these basic constructions enhances the novice draftsman's ability to draw accurately and to become more familiar with drafting instruments. Before beginning to draw polygons, you must understand the circumscribed and inscribed methods of drawing geometric constructions.

Circumscribed method

The *circumscribed method* of drawing polygonal constructions is a process by which the geometric figure is drawn inside a circle. The circle surrounds and defines the figure. The measurements for various planes or surfaces of the geometric figure are drawn *across the corners* or horizontal diameter of the circle.

Figure 2-32 is a square circumscribed by a circle.

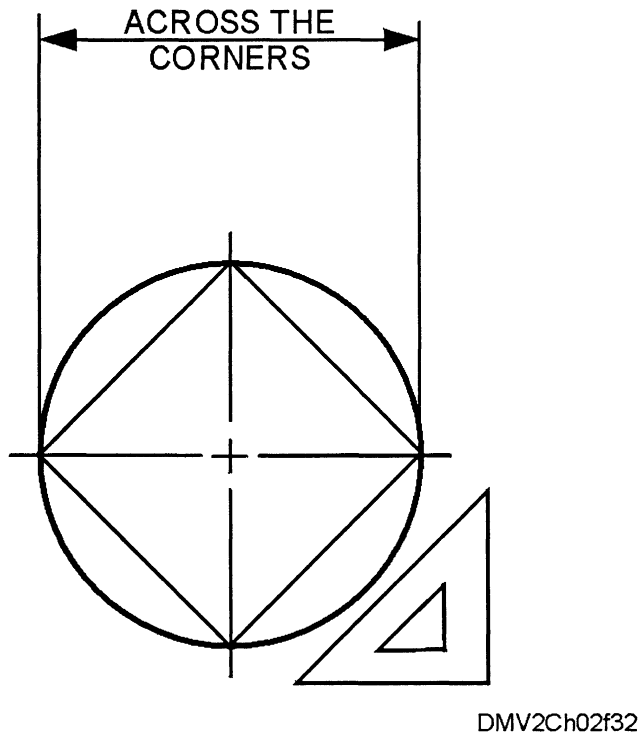

Figure 2-32.—A square circumscribed by a circle.

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Inscribed method

Drawing geometric figures around a circle is known as the *inscribed method* of polygonal construction. The circle is inside the figure and the sides of the geometric figure are tangential to the circle circumference. The diameter of the circle is measured at a 45° angle to the horizontal. Measurements for figure construction is made from the diameter or *across the flats.*

Figure 2-33 shows the construction of a square using the inscribed method.

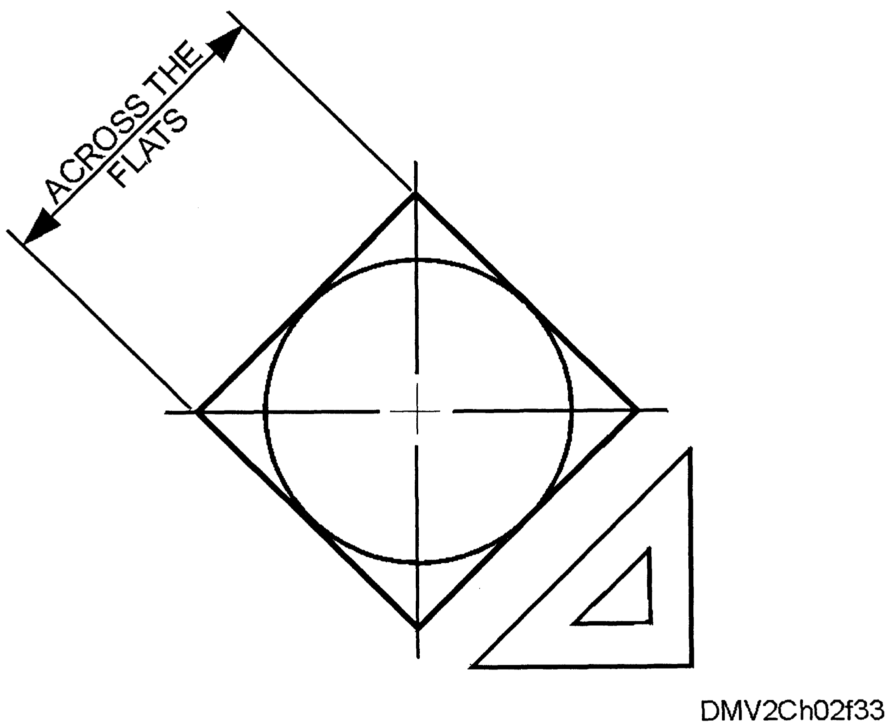

Figure 2-33.—A circle inscribed by a square.

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Angles

The most convenient method of laying out an angle is by using a triangle or a protractor. When absolute accuracy is important, use the tangent, sine, or chord method. A table of trigonometric tables listing tangent, sine, and chord values is located in the back of this book.

TANGENT METHOD: The *tangent method* of angle construction is a trigonometric function of an acute angle to find the ratio of the length of the side opposing the angle to the length of the side adjacent to the angle. In figure 2-34, the tangent of angle ∅ is *y/x*, and *y=x* tan ∅ . To construct the angle, assign a simple value to *x*, in this case 10. The larger the number, the more accurate the construction. Find the tangent of angle ∅ in a table of natural tangents, multiply by 10 and set off *y* = 10 tan ∅.

Figure 2-34 is an angle constructed by the tangent method.

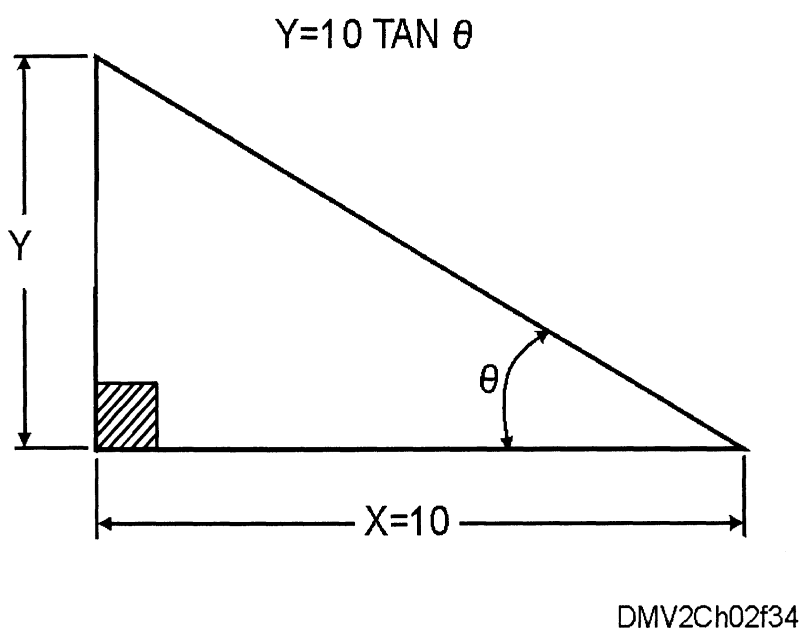

Figure 2-34.—The tangent method of angle construction.

Continued on next page

Angles (Continued)

SINE METHOD: The *sine method* of angle construction is another trigonometric function of the acute angle that is the ratio of the opposite side to the hypotenuse of a right triangle. Draw line *x* to any convenient length, again we will use 10. Find the sine of angle ∅ in a table of natural sines, multiply by 10, and strike *R*= 10 sin ∅. Draw the other side of the angle tangent to the arc.

Figure 2-35 is an angle drawn by the sine method.

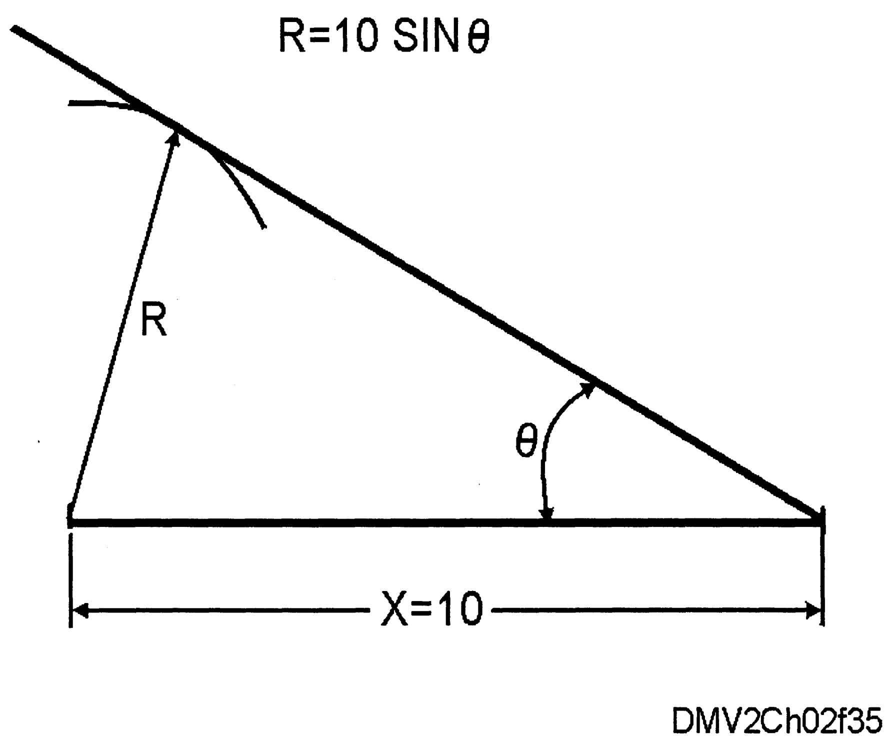

Figure 2-35.—The sine method of angle construction.

Continued on next page

Angles (Continued)

CHORD METHOD: The *chord method* of angle construction refers to the joining of two points on a curve by a line. Draw line x at any length and draw an arc at any radius, again let R= 10. Find chordal length C in the table of chords and multiply the value by 10. If a table is not available, calculate chordal value with the formula C = 2 sin ∅/2.

Figure 2-36 is an angle drawn by the chord method.

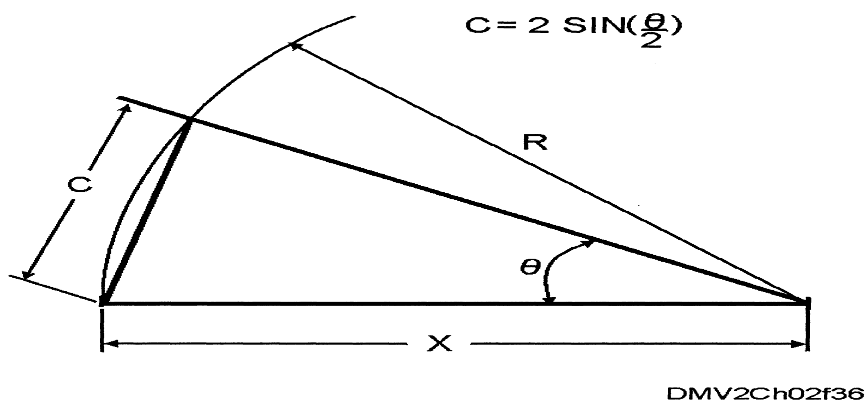

Figure 2-36.—The chord method of angle construction.

Continued on next page

Triangles

The preferred method of drawing triangles is with triangles or a protractor.

To draw triangles using triangles, use this table:

Step	Action
1	Select a triangle that contains the angles required to draw the triangle. For example, select a 30/60/90° triangle to draw an equilateral triangle where each angle equals 60°.
2	Draw a perpendicular line to the center of AB.
2	With given horizontal line AB, place the triangle base edge against the straightedge of your drafting table with the 60° angle at A.
3	Draw line AC to the perpendicular line.
4	Flip the triangle over to place the 60° angle at the base of the triangle at B and against the straightedge of your table.
5	Draw line BC. Lines AC and BC should intersect above the center of line AB and form a 60° angle to complete the equilateral triangle.

Figure 2-37 shows the construction of an equilateral triangle using a 30°/60°/90° triangle.

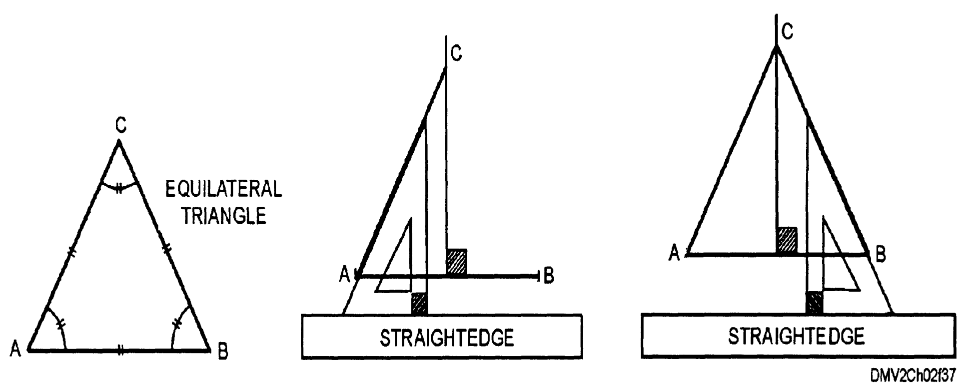

Figure 2-37.—Constructing an equilateral triangle.

Continued on next page

Triangles (Continued)

You may also use a protractor to construct triangles.

To use a protractor or semicircular protractor to construct an equilateral triangle, use this table:

Step	Action
1	With a straightedge, draw a line (AB) at any convenient length and a perpendicular line at the center.
2	Place the center of the protractor (usually marked 0) at A, align the protractor with the line AB.
3	Locate the 60° increment on the protractor and lightly mark your paper. If using a full circular protractor, you can increase accuracy by also locating and marking the opposing angle. This allows you to use the four-point reference system (*DM Volume 1,* chapter 2).
4	Move the protractor to B on line AB. Locate and mark the drawing at the 60° increment. Also mark the opposing angle , if possible.
5	Draw straight lines through the marked increments terminating at the perpendicular line. These lines should intersect forming an equilateral triangle.

Figure 2-38 shows an equilateral triangle constructed by using a protractor.

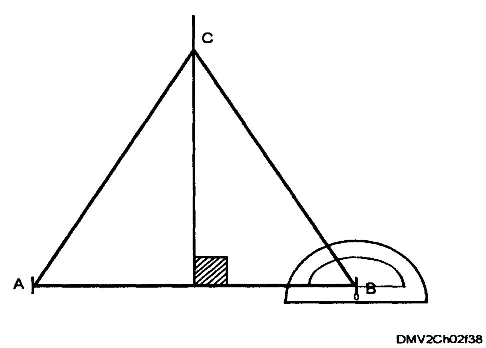

Figure 2-38.—Constructing an equilateral triangle with a protractor.

Continued on next page

Triangles (Continued)

You may use an alternate method of constructing triangles if triangles and protractors are not available.

To draw an equilateral triangle using an alternate method of construction, use this table:

Step	Action
1	Draw straight line AB the length of the base of your equilateral triangle.
2	Using the length of AB as a radius, strike an arc with A and with B as a center.
3	These arcs will intersect at C.
4	Draw straight lines from A to C and B to C to complete the triangle.

Figure 2-39 illustrates the process for constructing an equilateral triangle with an alternate method.

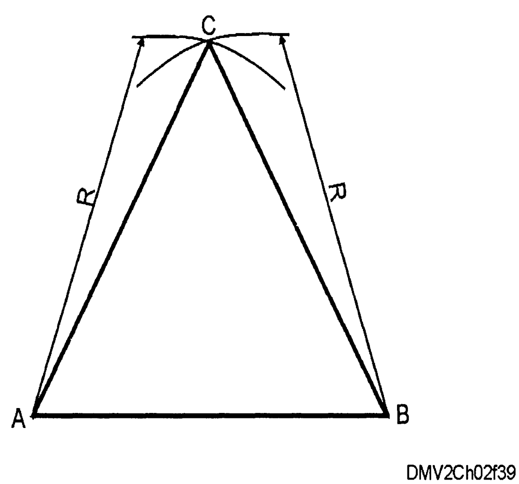

Figure 2-39.—Using a compass to construct an equalateral triangle.

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Squares

You may use triangles or protractors to construct squares. You may also construct a square using the circumscribed or inscribed method of construction.

To construct squares using the circumscribed method, use this table:

Step	Action
1	Draw horizontal and vertical lines intersecting at right angles to each other.
2	Using the intersection of these lines as a center, draw a circle of a diameter that equals the distance from one corner of the square to the opposing corner.
3	Where the circle intersects the horizontal and vertical centerlines, use a 45° triangle to draw connecting lines to form the square.

Figure 2-40 shows a square constructed using the circumscribed circle method.

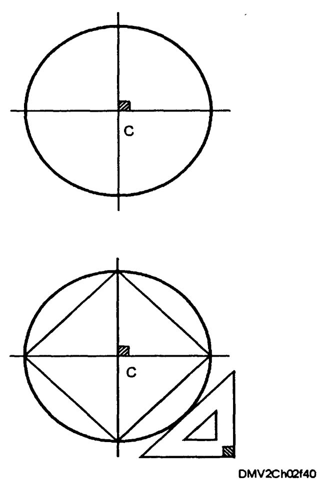

Figure 2-40. —Constructing a square using the circumscribed method.

Continued on next page

Squares (Continued)

To construct a square using the inscribed circle method, use this table:

Step	Action
1	Draw horizontal and vertical lines intersecting at right angles to each other.
2	Using the intersection of these lines as a centerline, draw a circle with a diameter equal to the distance between two opposing sides of the square.
3	Use a 45° triangle to draw straight lines tangent to the circumference of the circle and intersecting each other at 90°.

Figure 2-41 shows the construction of a square using an inscribed circle.

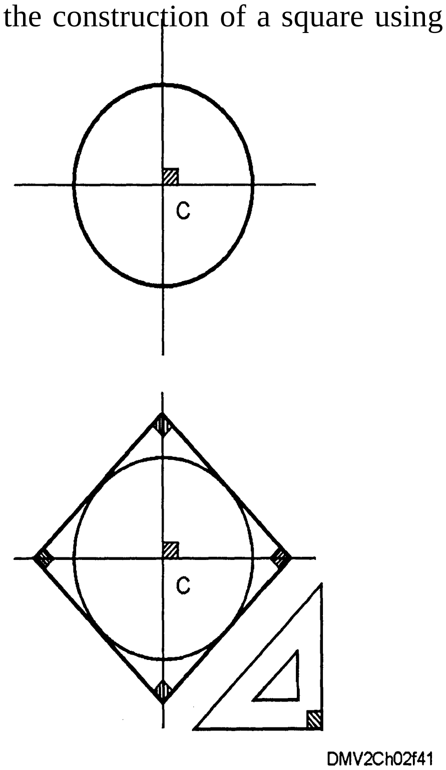

Figure 2-41.—Constructing a square using the inscribed method.

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Polygons composed of five or more sides

You can construct polygons composed of more than five sides using combinations of triangles, a pair of dividers, or a protractor. You may also circumscribe or inscribe multisided polygons in a square or circle. Always draw horizontal and vertical lines intersecting at right angles to each other first. Use the intersection of these lines as a center point for drawing circles or squares. The method for constructing multisided polygons with a pair of dividers is least accurate. The process requires you to estimate and lay out by trial and error equal portions along the circumference of the circle.

Figure 2-42 shows a pentagon constructed using a pair of dividers.

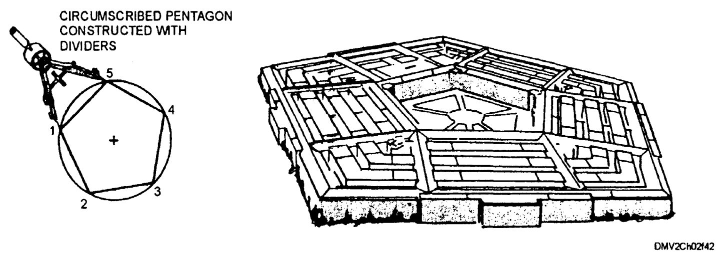

Figure 2-42.—The pentagon.

When using triangles to construct multisided polygons, you are limited to geometric figures that can be divided into angles that correspond to one of the 11 angles measurable by a triangle or combination of triangles such as hexagons (60°) and octagons (45°).

Figure 2-43 illustrates how to use triangles to construct a hexagon.

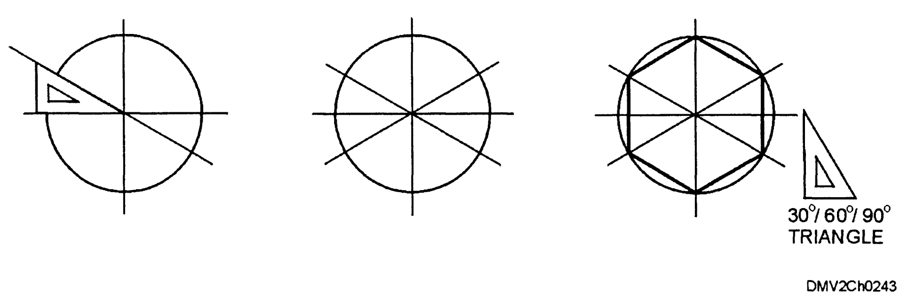

Figure 4-43.—A hexagon.

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Polygons composed of five or more sides (Continued)

Using a protractor to construct multisided polygonal figures requires you to mathematically compute the common angle using a formula. The formula divides the 360° of a circle by how ever many sides the polygon requires. For example, a heptagon or seven-sided polygonal figure requires a common angle of 51.3° (360° ÷ 7 = 51.3°).

Figure 2-44 shows a nine-sided (360° ÷ 9 = 40°) polygon.

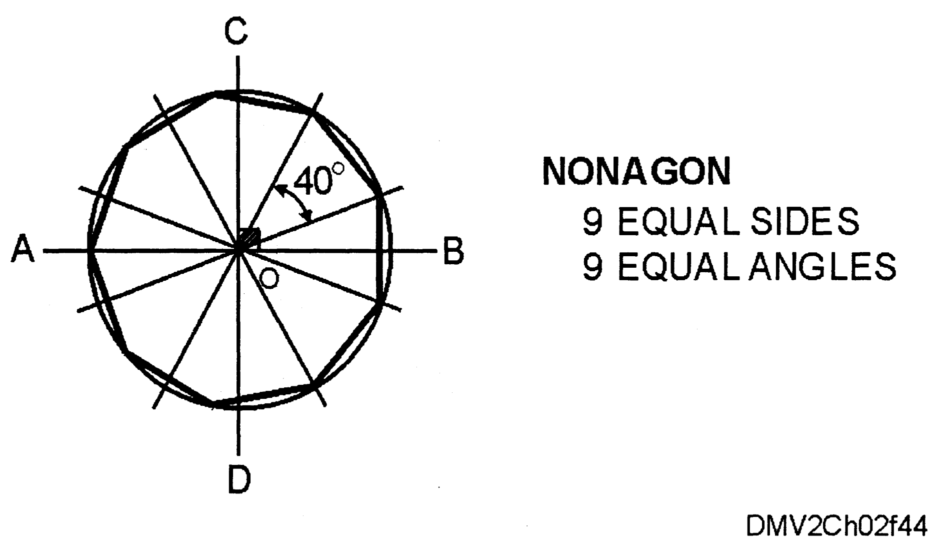

Figure 2-44.—A nongon.

Ellipse Construction

Introduction

Although ellipse templates greatly reduce construction time for an ellipse, you should know how to construct an ellipse using other methods. You may use the foci, trammel, concentric diameter, conjugate diameter, or circumscribed parallelogram method of ellipse construction.

Ellipses

An ellipse is created by moving a point so that the sum of its distances from two points (the foci) is constant and equal to the major axis. The foci serve as focal points for the rotation of the circumferential points. The major axis is its longest diameter. The shortest diameter is the minor axis. The ellipses formed is a basic, uniform, noncircular, closed curve. An ellipse may also be a conic section if formed by an oblique cutting plane on a solid.

Figure 2-45 shows ellipse terminology.

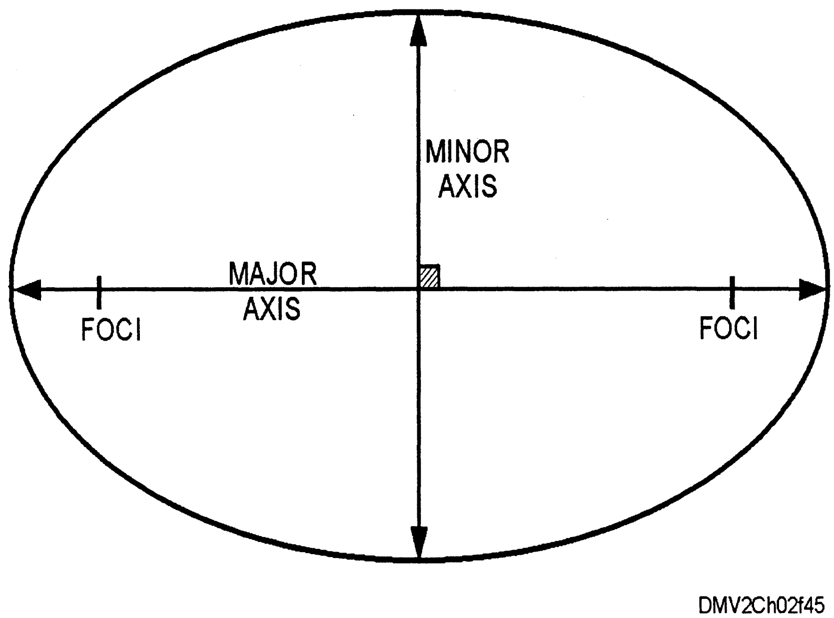

Figure 2-45.—Ellipse terminology.

Continued on next page

Determining foci

To determine the foci of an ellipse, strike arcs with a radius equal to half the major axis and with the center at the end of the minor axis. Another method is to draw a semicircle with the diameter equal to the major axis of the ellipse. Then draw GH parallel to the major axis. Draw GE and HF parallel to the minor axis.

Figure 2-46 show how to determine foci.

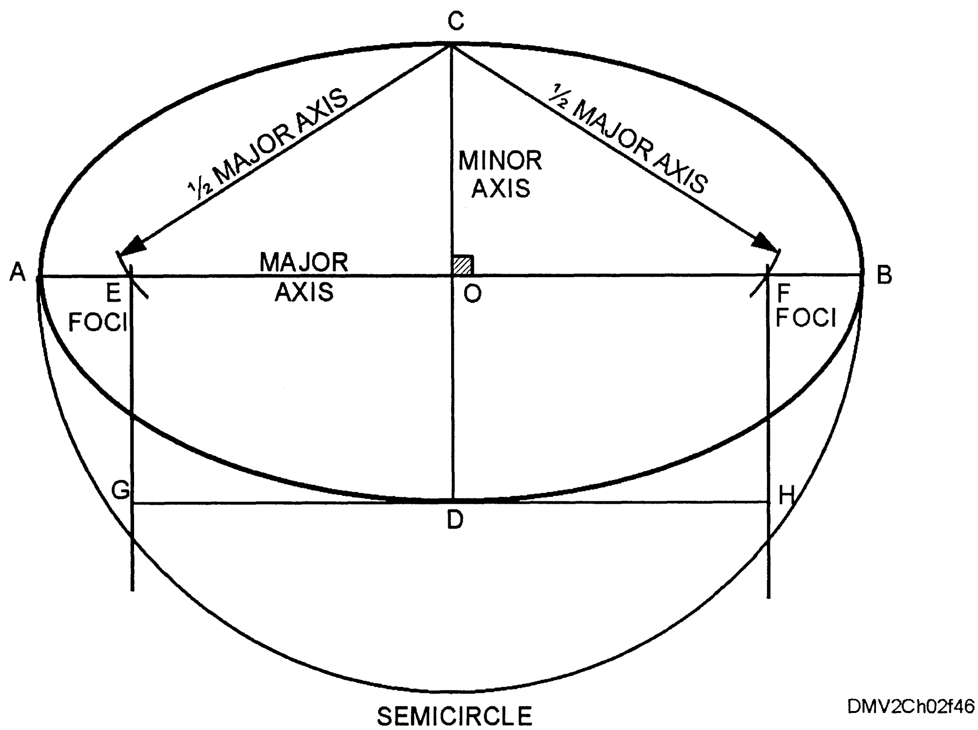

Figure 2-46.—Determining foci.

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Foci method

The *foci method* of ellipse construction involves plotting a series of points along the circumference of the ellipse by drawing a series of intersecting arcs using the foci on the major axis as centers.

To construct an ellipse using the foci method, use this table:

Step	Action
1	Lay out horizontal (AB) and vertical axes (CD) that intersect at right angles (0).
2	Locate the foci (F, F') by setting the compass to one half distance of the major axis AB and striking arcs along AB using C as the center.
3	Mark a minimum of five equal distances between F and 0. The more distances marked, the more accurate the ellipse construction.
4	Set the compass for the distance from A to 1. Strike arcs above and below line AB using 1 and F' as centers.
5	Set the compass for the distance A to 2. Strike arcs above and below AB using 2 and F' as centers. Continue plotting points this way until all five points form an ellipse circumference between CB and BD.
6	Mark a minimum of five equal distances between 0 and B.
7	Plot the five points forming an ellipse circumference between CA and AD using the same previous procedure but in reverse using F as the center.
8	Once all points are plotted, connect the points using french curves.

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Foci method (Continued)

Figure 2-47 illustrates the procedure for constructing an ellipse using the foci method.

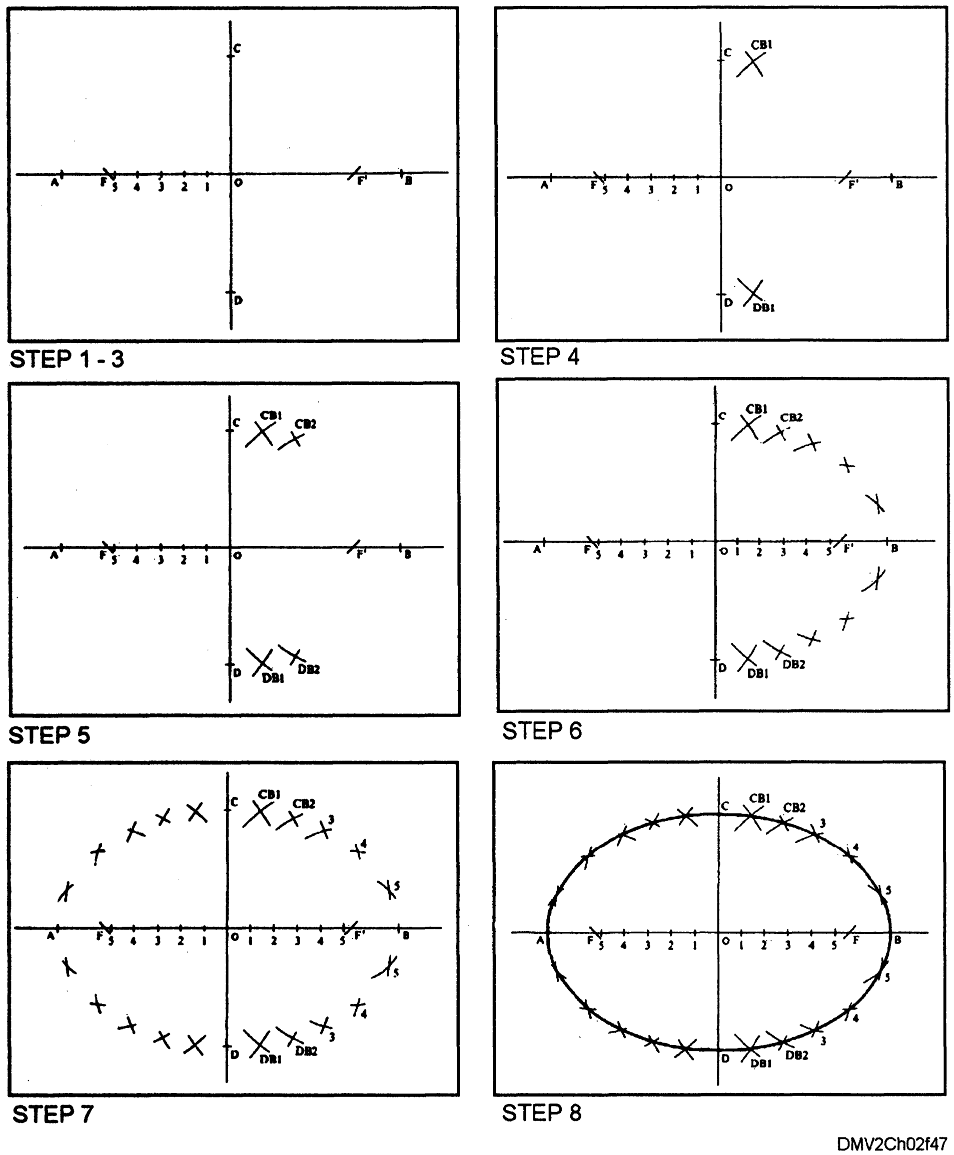

Figure 2-47.—Creating an ellipse by the foci method.

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The trammel method

The *trammel method* of ellipse construction involves plotting a series of points by using a strip of paper, cardboard, plastic, or straightedge marked with two foci and rotating the strip up, down, and around horizontal and vertical axes. The strip or length of paper or cardstock is a *trammel.* The trammel has three marks, two representing the foci and one representing the ellipse circumference.

To construct an ellipse using the trammel method, use this table:

Step	Action
1	Lay out horizontal (AB) and vertical (CD) axes that intersect at right angles (0).
2	Determine the minor and major axes and the foci of the intended ellipse.
3	On a strip of paper or cardstock, lay off distance GE representing half the length of the minor axis and GF representing half the length of the major axis.
4	Set the trammel on the drawing so that E is always traversing AB and F is moving along CD.
5	As you move the trammel, plot points at G which will always indicate the circumference of the ellipse.

Figure 2-48 shows the position of a trammel as you construct an ellipse.

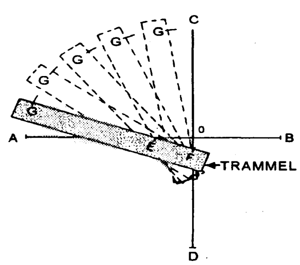

Figure 2-48.—The trammel method ellipse construction.

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Concentric diameters method

In the *concentric diameters method* of ellipse construction, you use the major and minor axes as diameters for concentric circles on a common horizontal and vertical axis intersecting at right angles. By drawing a diagonal across both circles and plotting subsequent points, you can construct an ellipse.

To construct an ellipse using the concentric diameters method, use this table:

Step	Action
1	Draw horizontal (AB) and vertical (CD) axes that intersect at right angles (0).
2	Using the length of the major and minor axes as diameters, lay out two concentric circles with 0 as a common center.
3	Draw a diagonal (XX) at any common angle through the circumferences of both circles passing through 0. Every diagonal drawn provides you with four points along the circumference of the ellipse.
4	From points X, draw lines XS parallel to CD and perpendicular to AB.
5	Where XX intersects the smaller circle, draw HE parallel to AB and perpendicular to CD.
6	Draw as many diagonals you feel necessary to adequately define the ellipse.
7	Lightly sketch the ellipse through the points. Darken the outline of the ellipse using french curves.

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Ellipse Construction, Continued

Conjugate diameter method (Continued)

Figure 2-49 illustrate the concentric circle method of ellipse construction.

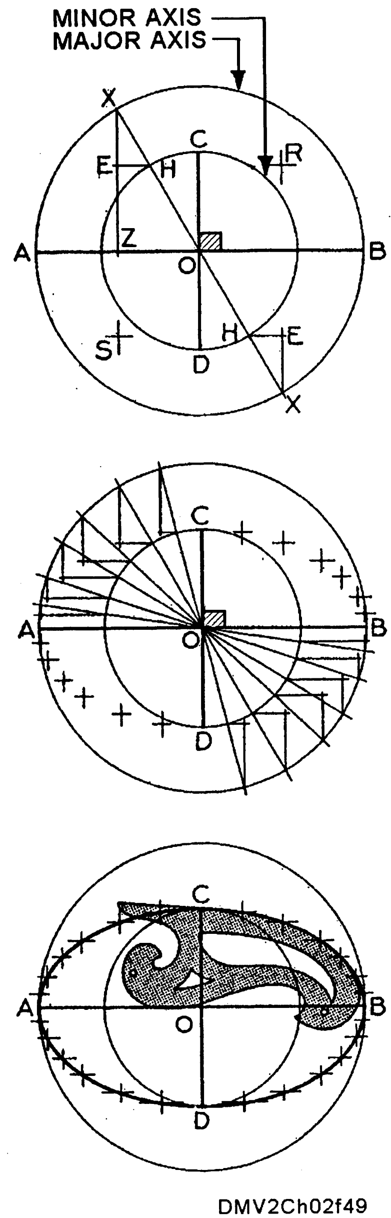

Figure 2-49.—Concentric circle method of ellipse construction.

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Conjugate diameter method

The *conjugate diameter method* of ellipse construction uses conjugate diameters to project a circle and through a series of tangents, plot points on the circumference of the ellipse.

To construct an ellipse using conjugate diameters, use this table;

Step	Action
1	Given two conjugate diameters (AB and DE) with C as a center, use the distance from C to A as a radius to draw a circle with C as the center.
2	Draw line (GF) perpendicular to AB and passing through C.
3	Draw lines connecting points D and F and points G and E.
4	Select any point (X) along AB and draw a line (PQ) parallel to DE and RS parallel to FG.
5	Determine at least five points to each quadrant. For larger ellipses, plot more points. The more points plotted the more accurate the ellipse circumference.
6	Lightly sketch the outline of the ellipse. Darken the ellipse using french curves.

Figure 2-50 illustrates the conjugate diameter method of ellipse construction.

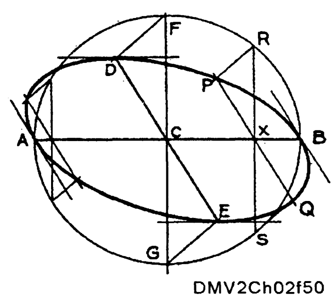

Figure 2-50.—The conjugate diameter method.

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Parallelogram ellipses

The *parallelogram method* of constructing ellipses inscribes the ellipse within a parallelogram. You may use conjugate diameters or the major and minor axes to formulate the parallelogram so long as the sides of the parallelogram are parallel to the diameters or axes.

To draw an ellipse by the parallelogram method, use this table;

Step	Action
1	Given the major and minor axes or the conjugate diameters AB and CD, draw a rectangle or parallelogram . Make sure all sides are parallel to their respective sides.
2	Divide the distance between AO and AJ into the same number of equal parts.
3	Starting at the ends of the minor axis CD, lightly draw straight lines through each point. The lines intersect forming the circumference of the ellipse.
4	Lightly sketch the outline of the ellipse. Darken the outline using french curves.

Figure 2-51 illustrates the procedure for drawing an ellipse inscribed within a parallelogram.

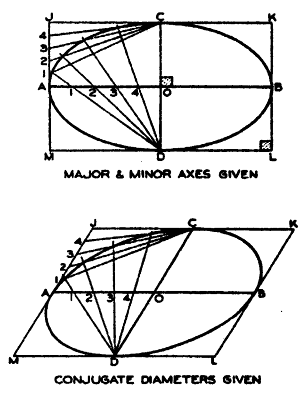

Figure 2-51.—The parallelogram method.

Involutes

Introduction

Some geometric figures are not bound by straight lines and arcs. They have no closed form but continue to spiral. This type of geometric figure is called an involute. Gear teeth and interlocking mechanisms are often depicted using this type of figure.

Involutes

An *involute* is the path of a point on a string as it unwinds from a line, polygon, or circle. Involutes are compound tangential arcs and semicircles of increasing larger diameters formed by lines, triangles, squares, and circles.

Involute of a line

To draw an involute of a line, use this table:

Step	Action
1	Given line AB, use line AB as a radius and B as a center to draw a semicircle AC.
2	Use AC as a radius and A as a center to draw another semicircle CD.
3	With BD as a radius and B as a center, draw semicircle DE.
4	Continue to repeat this pattern until the drawing is complete. Darken all outlines.

Figure 2-52 is an example of an involute of a line.

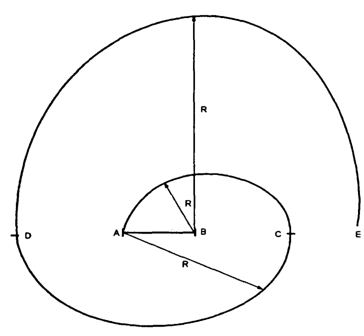

Figure 2-52.—The involute of a line.

Continued on next page

Involute of a triangle

To draw an involute of a triangle, use this table:

Step	Action
1	Given triangle ABC, extend the sides of the triangle to any convenient length.
2	Using CA as a radius and C as a center, strike arc AD terminating at the intersection of the extension BD.
3	With BD as a radius and B as a center, strike arc DE.
4	With AE as a radius and A as a center , strike arc EF.
5	Repeat this procedure until you reach a figure of the desired size.

Figure 2-53 is an example of an involute of a triangle.

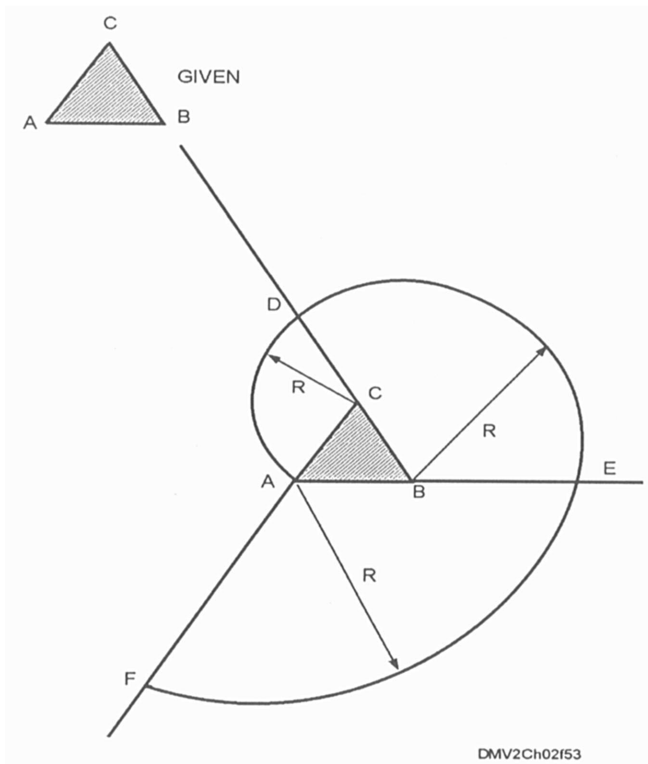

Figure 2-53.—The involute of a triangle.

Continued on next page

Involute of a square

To draw an involute of a square, use this table:

Step	Action
1	Given square ABCD, extend all sides any convenient length.
2	With CA as a radius and C as a center, draw arc AE.
3	With DE as a radius and D as a center, draw arc EF.
4	Repeat this procedure until you complete a figure of the desired size.

Figure 2-54 is an example of an involute of a square.

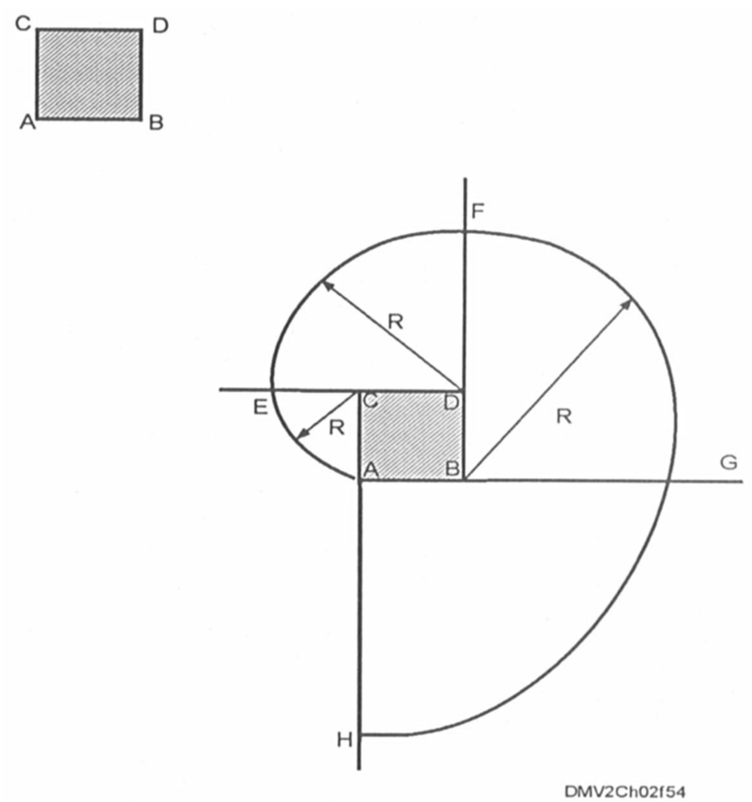

Figure 2-54.—An involute of a square.

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Summary, Continued

Review

This chapter begins with basic definitions of geometric figures. Explanations in the terminology and construction of two-dimensional figures such as lines, points, angles, triangles, quadrilaterals, and polygons form the foundation to understanding more the advanced geometric constructions of polyhedrons and solids. Bisection, division, transference, and tangency are all problem solving techniques that help you in advanced drafting situations. Polygonal, ellipse, and involute construction techniques should help to simplify geometric figure construction.

Comments

Geometric Construction is all about definitions. If you do not know the definitions in this chapter, study this chapter carefully and understand the terminology before you proceed into other chapters.

Once past the definitions, actual construction of geometric figures are simply, logically, and mathematically solvable. Geometric figure construction exercises your abilities to mentally solve problems and physically use drafting instruments to draw the solutions to problems on paper in a universally understood graphic language.

Geometric constructions are equally valid concepts whether you are drawing in pencil on paper or sitting at a computer drawing on a monitor screen.